Small Firm Financing and Dynamics

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Abstract

A growing body of work uses firm dynamics models in a variety of contexts to obtain results for firm behavior under the assumption of heterogeneous firms. A subset of these models add financial frictions, such that firms are subject to a collateral constraint limiting their borrowing threshold. In this paper, I consider the US Small Business Administration (SBA), estimating employment effects for firms receiving loans from its 7(a) or 504 programs and interpreting the mechanism of the SBA loan through the lens of a firm dynamics model with financial frictions. First, I measure the causal effect of loan disbursement on firm outcomes using a staggered adoption design. The program has positive effects particularly for small firms in the 7(a) program, and persistent effects even after the loan ends. Then, I present a firm dynamics model with financial frictions, and calibrate the model in a steady-state equilibrium. I implement the SBA loan in the model as an expansion of access to credit, and compare the model-implied labor response profile against empirical evidence. The counterfactual is qualitatively correct and matches patterns based on firm age yet does not match persistent effects of the loan found in the data. Alternatively, modeling the loan as an interest rate subsidy is more consistent with observed firm behavior.

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1 Introduction

A growing body of work uses firm dynamics models in a variety of contexts to obtain results for firm behavior under the assumption of heterogeneous firms. A subset of these models add financial frictions as a source of misallocation, such that firms are capital constrained (Hopenhayn (1992), Karahan et al. (2024), Ma et al. (2024), Buera et al. (2011), Midrigan and Xu (2014)). In the US, one of the goals of the Small Business Administration (SBA) is to increase access to capital for small firms facing credit constraints. In this paper, I estimate employment effects for firms receiving SBA loans and assess the extent to which existing models of firm dynamics with financial frictions can account for these measured effects.

The US SBA is the major case study of this paper. The SBA contains a set of programs and federal contracting services for the purpose of "[providing] counseling, capital, and contracting expertise as the nation's only go-to resource and voice for small businesses" and is the only such organization of its type in the country. Notably, its loan programs are designed to increase small business access to loans by reducing risk for the bank lender. In order to be eligible for the loan, businesses must fulfill a set of criteria, including being for-profit, physically residing in the US, being creditworthy, and having exhausted non-government financing options. The two main loan programs are the 7(a) and 504 loan programs: the 7(a) program is the primary program and the 504 program provides long-term, fixed rate financing for major fixed assets.[1] One advantage of studying the SBA is that it contains compiled information for all loans issued through the program.

What is the dynamic causal effect of loan receipt on firm-level employment? To what extent can the SBA loan be interpreted through the lens of a firm dynamics model with financial frictions? I first establish novel dynamic treatment estimates measuring the effect of loan receipt on employment, using heterogeneity-robust estimators and a staggered adoption design. I also document new patterns when splitting the sample by program, loan length, firm age, firm size, and firm loan-to-sales ratios. Then, I introduce a model of firm dynamics with financial frictions, extending Moll (2014). After calibrating the parameters and solving for the steady-state equilibrium, I simulate entering firms and execute a counterfactual exercise modeling loan disbursement as relaxing the collateral constraint to increase credit access. I compare the model-implied labor demand trajectory against the analogous empirical event study path. An additional counterfactual is done modeling the loan as reducing the price of capital instead.

[1] The 7(a) program encompasses several subprograms, such as Standard 7(a) loans, 7(a) Small loans, SBA Express, Export Express, CAPLines, Export Working Capital Program (EWCP), and Pilot Program loans. While loan requirements differ by subprogram, all loans are under $5 million in amount and subject to SBA interest rate limits.

Empirically, I find that firms receiving loans have increases in employment for up to 5 years afterwards, and that these effects are positive for loans in the 7(a) program. Within the 7(a) program, the effects are driven by small firms and are larger for firms that have higher loan-to-sales ratios.

Through the lens of the chosen model, I find that modeling the loan as relaxing the collateral constraint does not seem to be entirely consistent with the dynamic empirical estimates. The credit channel results in an increase in the constraint parameter which causes treated firm factor demand to spike immediately and drop upon restoration to the original. This contrasts the dynamic pattern of the empirical estimates, which documents a persistent effect on employment even after the loan ends. However, the channel correctly captures that younger firms have a larger treatment response than older firms. Alternatively, modeling the loan as a reduction in capital payment rates provides a closer match to the empirics. My results speak to both the fit of the primary mechanism of the chosen model in this specific empirical context, as well as the mechanism of the SBA loan through the lens of the chosen model.

This paper relates to several strands of literature. First, it adds to the existing theoretical and empirical literature on finance and growth. King and Levine (1993) and others use aggregate country-level data to explore the link between finance and growth. A standard identification problem has been the direction of causality between growth and finance. Recently, empirical papers have used firm-level data and novel identification strategies to estimate the impact of credit on firm growth, including Banerjee and Duflo (2014), which uses changes in firm size eligibility in Indian firms, and Bach (2014), which uses changes in sectoral eligibility in a French loan guarantee program.

Evaluating the SBA is an important policy evaluation task, as it is a government institution that has existed since the 1950s, and this paper builds on a few existing studies. The recent paper Agarwal et al. (2024) has evaluated SBA's Paycheck Protection Program (PPP) during the COVID-19 era, showing that the PPP fulfilled its original objective of relief to small businesses as well as reduced mortgage delinquency rates as a spillover effect. Brown and Earle (2017) conducts an empirical evaluation of the SBA 7(a) and 504 loan programs, finding that the program results in an increase of 3 to 3.5 jobs for each million dollars of loans. Orzechowski (2020) and Orzechowski (2024) both examine the relationship between the SBA lending program and state-level employment, finding a positive relationship between the growth in SBA lending and employment rates. Hurst and Pugsley (2011) shows that the majority of small businesses do not seek to innovate and are started due to non-pecuniary benefits.

Additionally, this paper builds on the firm dynamics literature, particularly with misallocation in the form of financial frictions. As reviewed in Hopenhayn (2014), reduced misallocation and selection of active firms are channels that have been used to study the

role of financial development for explaining developmental gaps. Hopenhayn and Rogerson (1993) is an early paper incorporating misallocation in the form of firing costs. In addition, several papers consider the quantitative impact of financial constraints on output measures such as GDP per capita, TFP, and the capital-output ratio, notably Buera et al. (2011), Midrigan and Xu (2014), and Gopinath et al. (2017). Papers focusing on start-up dynamics use the Hopenhayn firm model framework, such as Karahan et al. (2024) and Ma et al. (2024). Other papers use the span-of-control model in Lucas Jr. (1978), such as Garicano et al. (2016).

Lastly, this paper is related to the empirical literature on firm financing. Borrowing constraints are typically modeled as resulting from either cash flows from the firm's operations or from liquidation value of physical assets. Lian and Ma (2021) finds that in the US, 80% of corporate debt is based on the value of cash flows from firms' continuing operations. However, this result is reversed for small, young, and low profit-firms: the median share of cash flow-based lending is 8% and of asset-based lending is 53%. The collateral constraint form is thus a suitable assumption for SBA firms. Aydin and Kim (2025) is a recent paper that identifies firm precautionary motives as a reason limiting their growth even with low default risk.

This paper makes several notable contributions to the literature. First of all, I find support for the empirical evidence offered in Brown and Earle (2017) using a new methodology accounting for heterogeneous treatment effects, on a more recent sample of firms, and several new dimensions of firm characteristics. Second of all, I extend Moll (2014) to a small business setting and develop novel counterfactual exercises to test the mechanism of loan receipt, finding that SBA loans do not seem to function as relaxing the collateral constraint to increase credit supply, but rather function more as a reduction in cost of capital. I also find that there are differential effects of the mechanism of the loan depending on the firm's wealth level at the time of receipt. In the absence of the relevant data and of previous structural work testing the collateral constraint channel in an empirical firm-level context, these results help to better understand the constraints facing firms receiving SBA loans and the fit of the chosen model within the context of a real loan program.

This paper proceeds as follows. Section 2 presents a motivating example. Section 3 provides institutional context and descriptive statistics of the SBA loan program and firm characteristics. Section 4 provides empirical results aiming to measure the causal effect of loan receipt on firm characteristics. Section 5 presents the model and subsequent simulations. Section 6 presents exercises interpreting effects of the SBA program through the lens of the model. Section 7 considers the robustness of results to model modifications. Section 8 concludes. Section A contains all sections of the Appendix.

2 Motivating Example

Before introducing the empirical setting for this paper, I begin by presenting a simple case of an entrepreneur who owns a firm and is subject to credit constraints. This example is introduced to provide intuition for the effect of the SBA loan in a structural context if it is interpreted as a relaxation of the collateral constraint. After introducing a simple setting, I will study changes in the firm's labor demand after providing them with more credit. There will be two scenarios: one where the collateral constraint is permanently relaxed, and another where the collateral constraint is temporarily relaxed. The latter is designed to be more representative of a loan.

The entrepreneur maximizes their lifetime expected utility:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_t \right]$$

The firm produces with decreasing returns to scale production:

$$y = f(z, k, l) = (zk)^{\alpha} l^{\gamma}$$

and faces the following collateral constraint:

$$k_t \leq \lambda a_t, \ \lambda \geq 1$$

Suppose that the entrepreneur has a fixed productivity level z and starts their life with a fixed wealth level a.

Define profit function:

$$\Pi(a, z) = \max_{k,l} \{ f(z, k, l) - wl - (r + \delta)k \} \text{ s.t. } k \leq \lambda a$$

and the evolution of wealth:

$$a_{t+1} - a_t = \Pi(a_t, z_t) + (1 + r_t)a_t - c_t$$

The firm rents capital at rate $r + \delta$ and hires workers at wage w.

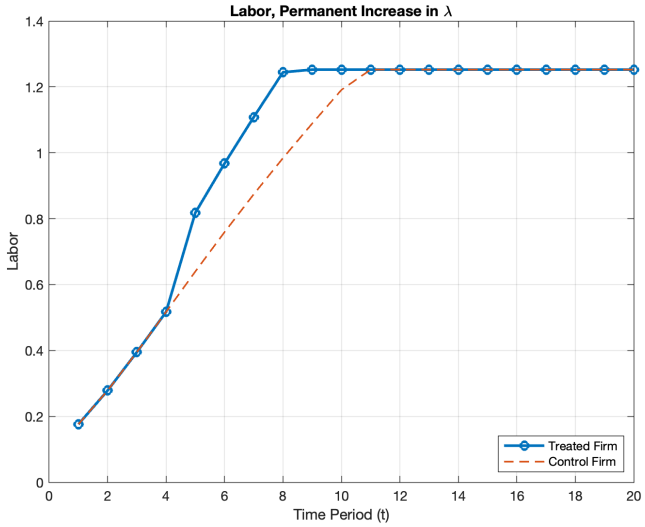
Table 1: Parameter Values

Parameters	Values
Capital share (α)	0.3
Labor share (γ)	0.55
Discount rate (β)	0.95
Depreciation rate (δ)	0.05
Collateral Constraint (λ)	4.15
Wage (w)	0.76
Interest rate (r)	0.08
Productivity level (z)	1
Initial wealth (a_0)	0.05

Under this framework, suppose that the firm receives a loan, functioning as a relaxation of the collateral constraint. Table 1 presents the parameter values for this simple example. The optimal policy function a_{t+1} is solved numerically.

Figures 1 presents factor demands for the baseline firm calibrated to Table 1 values and with a permanent increase in λ. As can be seen in the figure, the firm grows more quickly on the treated path, and becomes unconstrained at T=11. To observe the extent to which their labor response depends on their characteristics, I then present results for firms both varying their starting productivity level (with wealth fixed) and varying their starting wealth level (with productivity fixed), as wealth and productivity are the two dimensions of heterogeneity in the model. Figure 2 presents the results varying productivity and wealth respectively. For both of these cases, the extra credit allows the firm to grow faster and reach its target size sooner. Figure 3 provides corresponding visuals comparing the percentage increase in labor for the firm following its treated path versus untreated path. Firms with higher productivity and lower wealth levels have a greater treatment response after collateral relaxation.

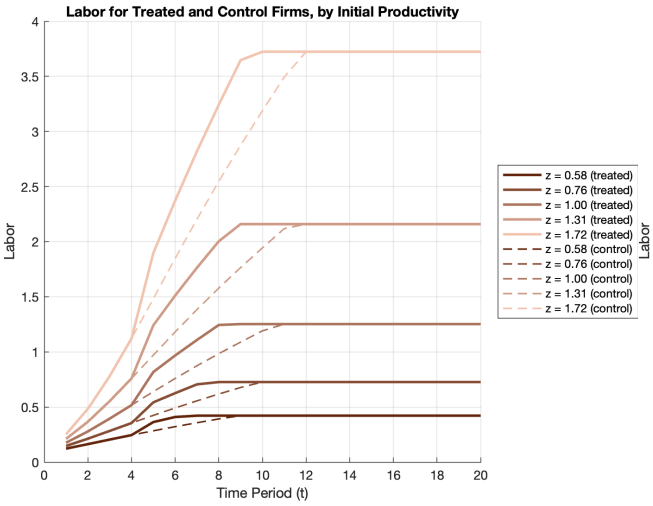
Baseline Factor Demands



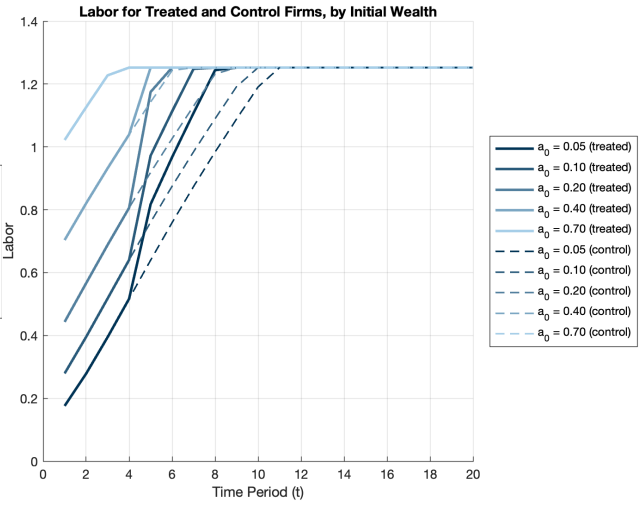
Note: This figure plots outcomes over T=20 for one simulated firm.

Permanent Counterfactual Factor Demands

(a) Varying Productivity



(b) Varying Wealth

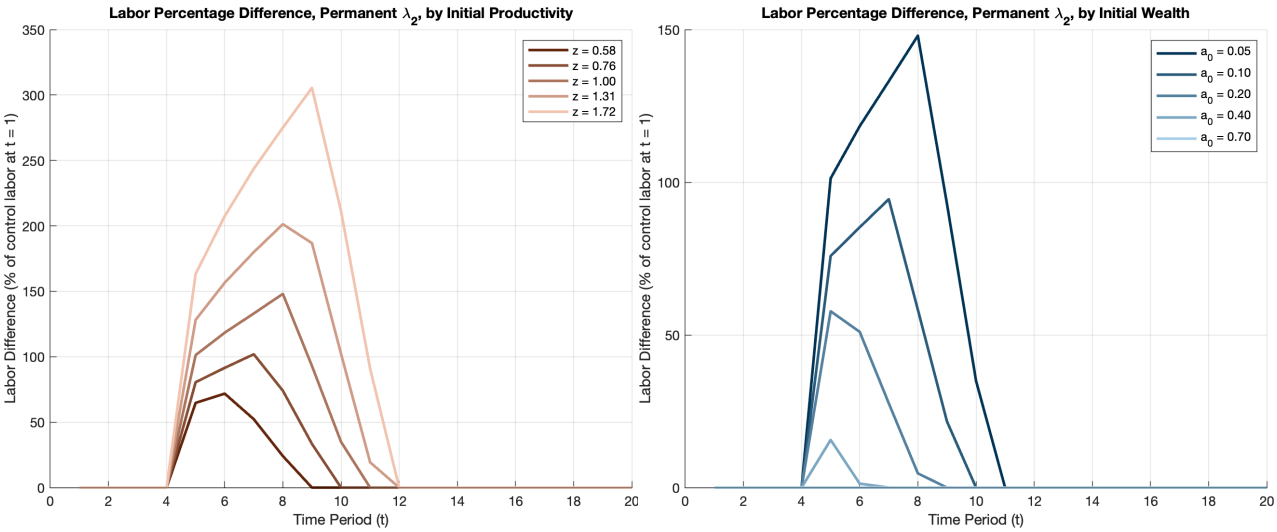


Note: This figure plots outcomes over T=20 for one simulated firm across varying wealth and productivity levels. It receives a counterfactual at T=4 permanently.

Permanent Counterfactual Labor Percentage Difference

(a) Varying Productivity

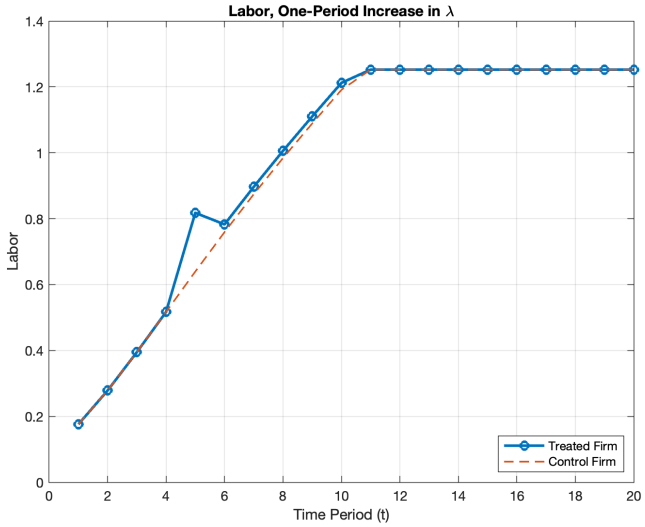
(b) Varying Wealth



Note: This figure plots outcomes over T=20 for one simulated firm across varying wealth and productivity levels. It receives a counterfactual at T=4 permanently.

Next, I present labor demand for a firm with a temporary increase in λ in Figure 4. As before, I then vary the firm's starting productivity and wealth levels to evaluate the impact of firm heterogeneity on their treatment response, presented in Figures 5 and 6. As before, higher productivity and lower wealth starting levels result in greater impact of the constraint. Due to the temporary nature of the counterfactual, there is a sharp increase of factor demands which decreases sharply upon reinstatement of the initial value, while still remaining larger than if the counterfactual was not received. The firm with the relaxed constraint has around 100% more labor than the equivalent non-treated firm during the loan, compared to an around 15% more labor than the equivalent non-treated firm one period after the loan.

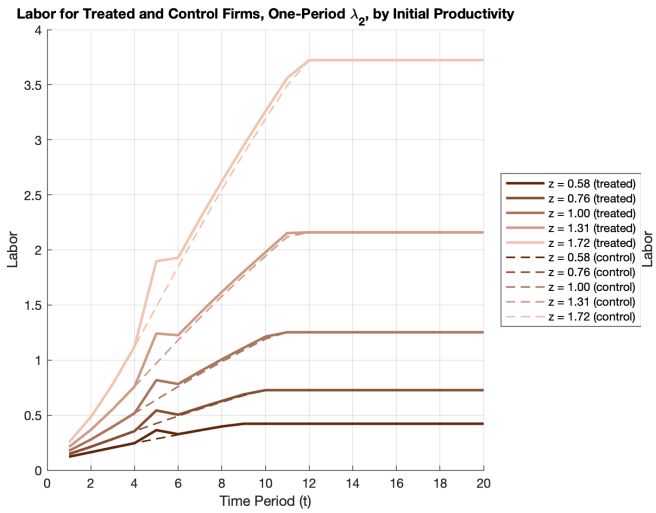
Baseline Factor Demands



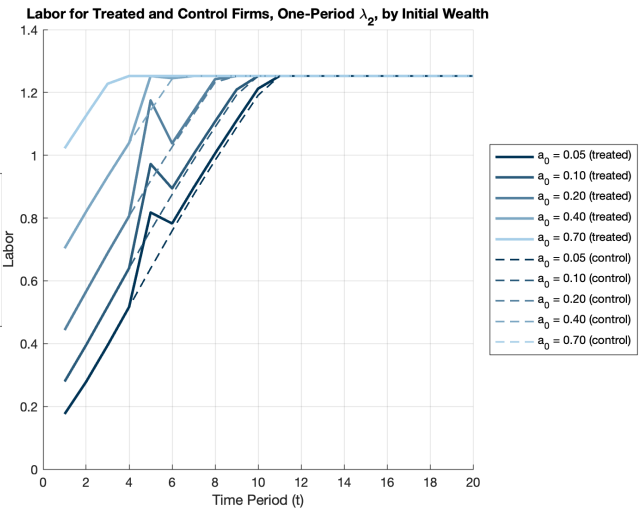
Note: This figure plots outcomes over T=20 for one simulated firm.

Temporary Counterfactual Factor Demands

(a) Varying Productivity



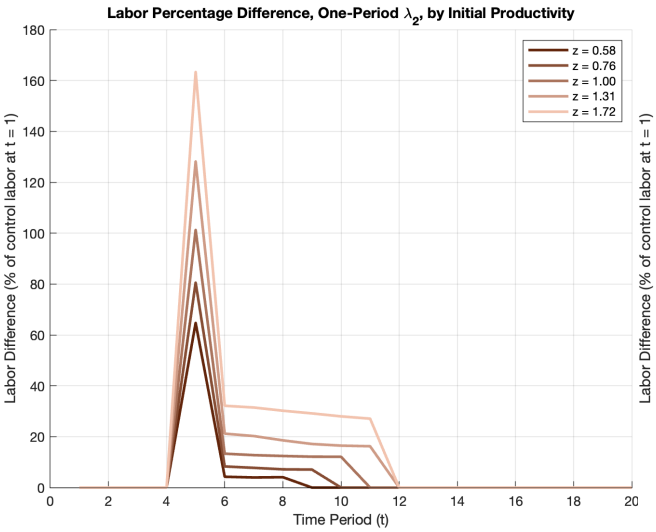
(b) Varying Wealth



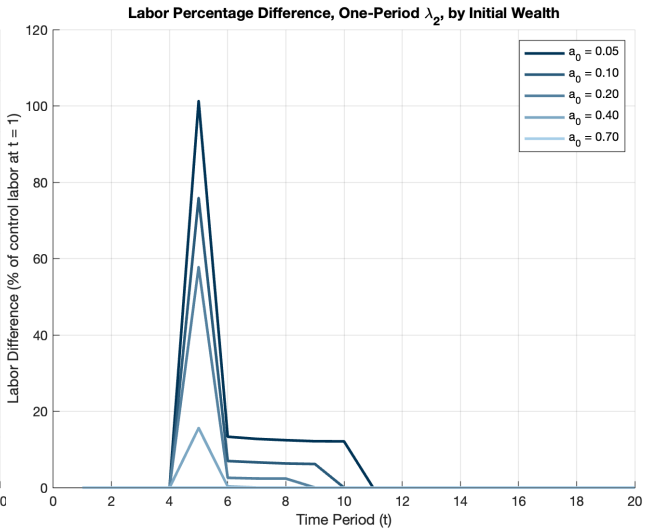
Note: This figure plots outcomes over T=20 for one simulated firm varying wealth and productivity levels. It receives a counterfactual at T=4 for 1 period.

Temporary Counterfactual Labor Percentage Difference

(a) Varying Productivity



(b) Varying Wealth



Note: This figure plots outcomes over T=20 for one simulated firm varying wealth and productivity levels. It receives a counterfactual at T=4 for 1 period.

There are two key takeaways from this exercise. First, firms grow faster when the the collateral constraint is relaxed and they are allowed to borrow more. Second, the size of the firm's labor response depends on their wealth and productivity levels. The second takeaway provides evidence that there are heterogeneous treatment responses in the model, making it a reasonable concern for empirical work as well.

3 Institutional Context and Descriptive Evidence

The SBA contains a set of programs and contracting services for the purpose of providing resources for small businesses. Its loan programs are designed to increase small business access to loans by reducing risk for the bank lender. To be eligible for the loan, businesses must fulfill a list of criteria. One of these criteria is the size standard policy: the maximum number of employees or annual receipts of the firm that qualifies it to be a small business eligible for SBA programs.[2] In addition, it must fulfill a Credit Not Available Elsewhere Clause, that the firm is unable to obtain the requested funds from a non-government source on reasonable terms. Notably, this clause does not make a distinction between a firm who does not have access to any funds or a firm who does have access to funds, but on unreasonable terms.

[2]Generally, the size standard policy is on the order of $1 to $50 million for industries where the policy is in firm net worth, and 100-1,500 employees for industries where the policy is in number of employees.

The two main loan programs are the 7(a) and 504 loan programs. The 7(a) program is the most versatile program and used for general business needs, and the 504 program is best used for long-term fixed assets. Figure 7 displays a diagram of the SBA institution, showing that the program loans are conducted in conjunction with a lending partner. Since the lender receives a loan guarantee from the government, they are more incentivized to lend to small businesses which may have limited credit history or capital.

In general, the loan terms can vary widely. Loan amounts can range from $5000 to $5 million. Disbursement may be lump-sum or as incremental payments, and the full disbursement process occurs within 4 years of the approval date. Interest rates are negotiated between the borrower and lender. The repayment period has a maximum of 7 years for working capital, 10 years for equipment or inventory, 25 years for real estate. The loans are fully amortized, so they are paid off over a set period through regular, fixed payment installments.[3] In the case of default, collateral is seized, and eventually government takes responsibility for debt collection.

I use loan data from the 7(a) and 504 programs, from years 2011 to 2018, available under the Freedom of Information Act (FOIA). The data contain information on the business, the bank, loan amount, interest rate, loan status, and loan program. I match these loans to firm data from the Dun and Bradstreet (D&B) datasets, which contains annual establishment-level information including establishment employment, sales, industry, and geography. Matching was conducted based on establishment name and address. To convert nominal dollars into real dollars for the D&B sales data, I use the annual Consumer Price Index (CPI) as published by the Federal Reserve Bank of Minneapolis, and normalize to dollar values in 2019. At the end of this procedure, the dataset contains 96,925 unique firms with loans. Within this sample of firms, 88,895 received loans from the 7(a) program, and 8,030 received loans from the 504 program. The unit of analysis is the establishment, although the terms firm and establishment are used interchangeably throughout the paper. I restrict attention to the sample of firms which paid their loan in full. Additionally, I drop firms which have 0 sales and 0 employment listed in their event year, and impute 1 as the minimum employment value for all remaining firms with 0 listed employment.

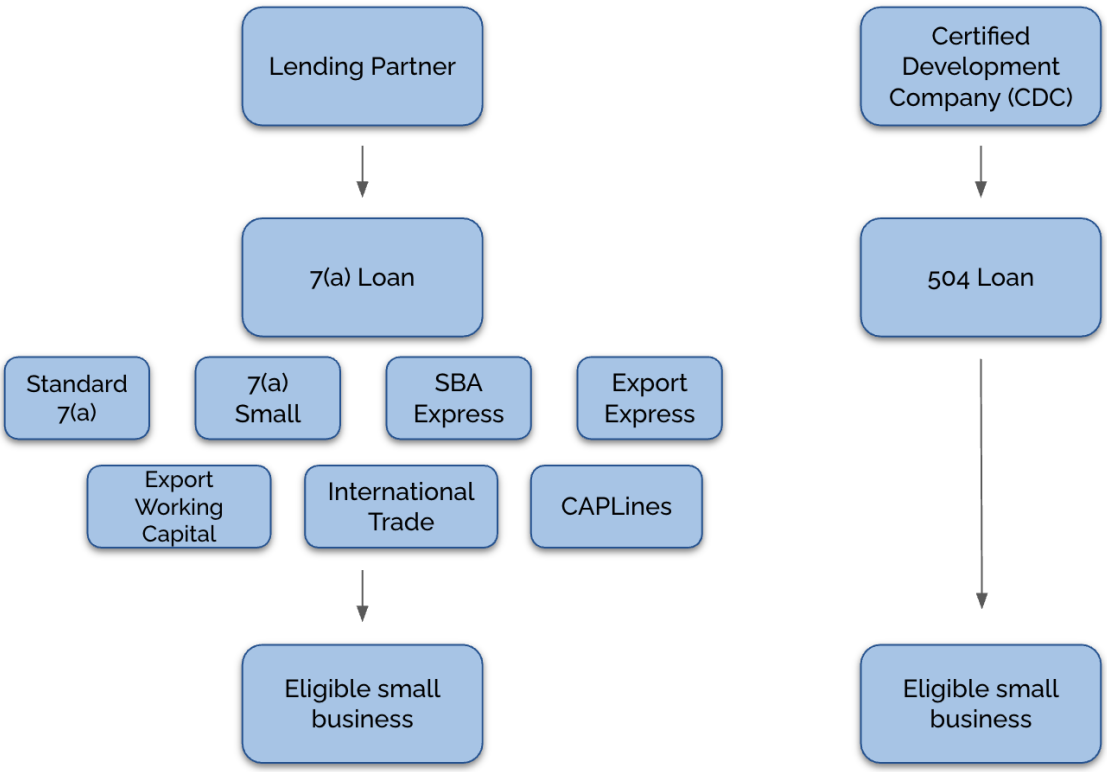
Table 2 presents summary statistics for matched firms receiving loans from either the 7(a) or 504 program over the decade time interval. Firms receiving loans from the 504 program report having higher loan amount, greater employment at the time of loan receipt, and more sales at the time of loan receipt, reflecting differences in the two programs. Since the 504 program targets long-term financing for major fixed assets and has a higher maximum borrowing amount, firms that receive 504 loans tend to be larger and more profitable at the time of loan receipt.

[3] More of the payment goes toward interest in the beginning, and toward the principal balance in the end.

Figure 8 presents the kernel density distribution of firm size and log sales for the same sample of firms. The employment graph shows that most firms have less than 10 employees, which is consistent with the average employment values presented in Table 2. The log sales graph shows that firms with non-zero sales values have a sales value on the order of $100,000.

In the next section, I will consider treatment heterogeneity by event year cohorts of firms. Therefore, Figure 9 presents kernel density distributions of employment, sales, age, and loan amount by treatment cohort. Across cohorts, most establishments have employment counts of 10 workers or under, sales in the six figure range, have age of 10 years of less, and borrow a loan of amount $400,000 or less.

SBA Institutional Diagram for Major Loan Programs



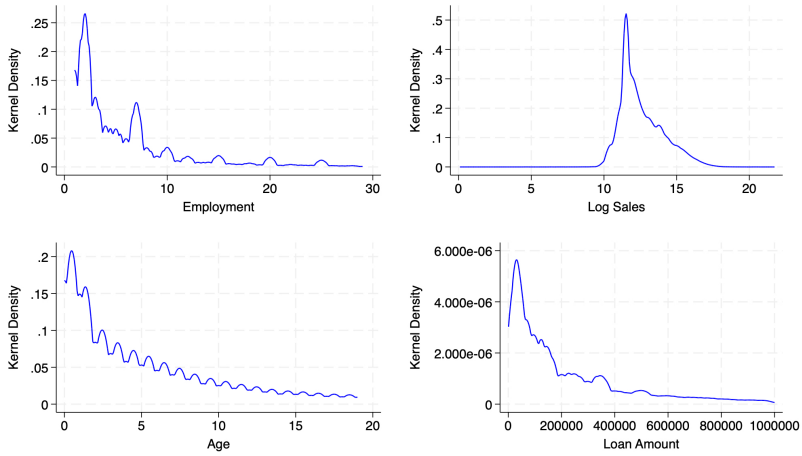
Note: This figure documents the institutional context for the SBA 7a and 504 loan programs. The loan programs are conducted in conjunction with a lending partner and certified development company (CDC), respectively. This diagram does not include other funding programs of the SBA, namely the investment capital, disaster assistance, surety bonds, and grants programs.

Table 2: SBA Loan Data Summary Statistics

7(a)	Mean	SD	Median	Min	Max	N
Loan Amount	342731.01	659155.40	100000.00	1000.00	5000000.00	88895.00
Interest Rate	6.43	1.54	6.00	0.00	11.50	88895.00
Loan Length	4.25	2.28	4.00	0.00	13.00	88883.00
Age	7.75	10.90	4.00	0.00	183.00	86641.00
Employment	8.03	18.97	3.00	1.00	1500.00	88895.00
Ln Sales	12.53	1.44	12.14	0.09	21.73	88011.00
504	Mean	SD	Median	Min	Max	N
Loan Amount	778802.37	829829.30	494000.00	29000.00	5500000.00	8030.00
Loan Length	6.07	2.55	6.00	0.00	13.00	8030.00
Age	11.73	12.93	8.00	0.00	132.00	7578.00
Employment	17.64	67.94	7.00	1.00	3500.00	8030.00
Ln Sales	13.46	1.72	13.26	9.69	20.48	7760.00

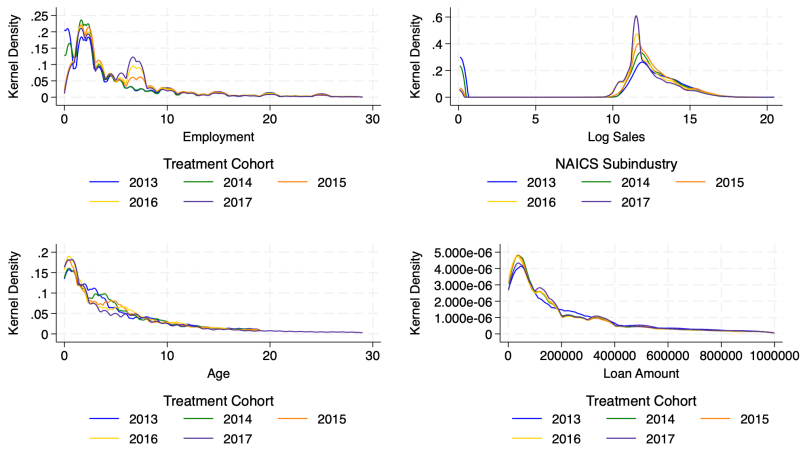
Note: This table contains summary statistics for matched SBA loans during the 2011-2018 period during the year of loan receipt. Loans are matched to their corresponding firm in Dun and Bradstreet data using business name and state after systematic string cleaning. In the event of duplicates, the street address with the highest probabilistic score was used. Interest rates are not reported for 504 program loans.

Figure: Firm Characteristics



Note: This figure plots kernel density distributions of firm employment, log sales, age, and loan amount for the same sample of firms as in Table 2. Only firm observations in the year of loan receipt are included. The epanechnikov kernel estimator was used.

Figure: Firm Characteristics by Treatment Cohort



Note: This figure plots kernel density distributions of firm employment, log sales, age, and loan amount for firms by treatment cohort. This figure uses the same sample of firms as in Table 2. Firms with sales values of 0 were imputed with sales values of 1 in order to take the logarithm of the variable. Only firm observations in the year of loan receipt are included. The epanechnikov kernel estimator was used.

4 Staggered Adoption Results of Loan Receipt

In this section, I review recent results from the literature and present an area of econometric concern. Then, I present my methodology, estimators used, and the set of empirical results. Within my results, I consider the sample of firms as introduced in Section 3, as well as sub-samples by program, firm characteristics, and loan length.

4.1 Previous Literature and Contribution

In the existing literature, Brown and Earle (2017) provide empirical estimates on the causal effect of an SBA loan on firm employment. Their IV regression results suggest that there is a steady upward trend relative to control firms up to 5 years after loan receipt, and they conclude that "$1 million of loans raises loan recipient employment by 3–3.5 jobs on average during the first three postloan years".[4] To gain a sense of how this result scales to the average firm, consider the mean SBA loan amount reported in Table I of their paper, which ranges from $286,288 to $297,010 depending on their provider bank. Thus, the result for $1 million in loans translates to an increase in 0.89 to 1.04 jobs on average during the first three postloan years for a firm with the average loan.

The ordinary least squares (OLS) results provided in Brown and Earle (2017) follow a two-way fixed effects estimator with a staggered adoption design and a matched differences-in-differences procedure:

$$E_{it} = \rho_t + \alpha_{j(i)} + X_{it}\delta + D_{it}\beta_{fe} + \epsilon_{it} \tag{1}$$

where E is the firm outcome, i indexes firms, $j(i)$ indexes pairs of matched treated and control firms, X_{it} is a vector of controls, and t indexes years. ρ_t contains fixed effects by year and α_j contains fixed effects by treatment-control group. β_{fe} is the effect of interest.

A major issue with estimating β_{fe} with OLS is that it does not correctly weight group-time average treatment effects (ATEs), as described by Chaisemartin and D'Haultfoeuille (2020) and various other papers. Let g represent treatment-control firm groups in the equation above, $D_{g,t}$ represent the treatment indicator by group and time, and $W_{g,t}$ represent weights on each (g,t) group. Then β_{fe} is defined as:

$$\beta_{fe} = \mathbb{E}[\sum_{(g,t):D_{g,t}=1} W_{g,t}\Delta_{g,t}]$$

Although the weights $W_{g,t}$ sum to 1, some of them may be negative if homogeneous treatment effects do not hold, biasing the result for β_{fe}. Moreover, in this paper, homo-

[4]The outcome variable is number of employees per $1 million loan, which differs from firm outcomes used in this paper. Whereas our results are interpreted by firm, their results are interpreted by loan amount.

geneous treatment effects might not be assumed to hold over time or across groups. For instance, due to fluctuations in aggregate credit conditions each year, differences in composition of firms for treatment cohorts, and fluctuations in the circulation of information concerning the SBA programs, there may be expected to be heterogeneous treatment responses. Additionally, in Section 2, the model predicts that heterogeneous firms will have heterogeneous treatment effects. Thus, heterogeneity-robust TWFE estimators are applied to obtain estimates of β_{fe} allowing for treatment effect heterogeneity.

Diagnostic test results from Chaisemartin and D'Haultfoeuille (2020) provide additional evidence for the expected improvement using heterogeneity-robust estimators compared to TWFE estimators. As seen in Table 3, the estimate for the diagnostic test statistic $\underline{\sigma}_{fe}$ is 0.0585. This estimate is sufficiently close to 0 which implies that treatment effect heterogeneity is a valid concern.

Table 3: Robustness Measures for Heterogeneous Treatment Effects

Estimator	Value	Uniform	Normal
$\underline{\sigma}_{fe}$	0.0585	[-1.0132, 1.0132]	[-1.1466, 1.1466]

Note: This table computes $\underline{\sigma}_{fe}$ as defined in Corollary 1 of Chaisemartin and D'Haultfoeuille (2020). The value of $\underline{\sigma}_{fe}$ is 0.1123. If $\underline{\sigma}_{fe}$ and $\underline{\sigma}_{fe}$ are close to 0, this means that treatment effect heterogeneity would be a serious concern for the validity of $\hat{\beta}_{fe}$. The two columns on the right provide intervals, assuming the given distribution of (g,t) cells treatment effects, that are used to evaluate if the estimator value is considered high or low.

This paper builds upon the OLS results with the following additions: (i) a more recent sample of firms, (ii) applying new techniques to the staggered adoption design, and (iii) splitting the sample by several dimensions of firm characteristics.

4.2 Methodology

I estimate the causal effect of loan receipt on firm outcomes using the heterogeneity robust two-way fixed effects estimators in Borusyak et al. (2024), Callaway and Sant'Anna (2021), and Chaisemartin and D'Haultfoeuille (2020).[5]

The estimator in Borusyak et al. (2024) is defined using the following imputation representation of the efficient estimator:

1. Within the untreated observations ($it \in \Omega_0$), estimate λ_i and δ by OLS in the regression $Y_{it} = A'_{it}\lambda_i + X'_{it}\delta + \varepsilon_{it}$

[5]To apply the estimators, I use the statistical packages provided by the papers: csdid, did_imputation, and did_multiplegt respectively.

2. For each treated observation ($it \in \Omega_1$) with $w_{it} \neq 0$, set $\tilde{Y}_{it}(0) = A'_{it}\tilde{\lambda}^*_i + X'_{it}\tilde{\delta}^*$ and $\tilde{\tau}^*_{it} = Y_{it} - \tilde{Y}_{it}(0)$ to obtain the estimate of the causal effect τ_{it}

3. Estimate the target τ_w by a weighted sum $\tilde{\tau}^*_w = \Sigma_{it\in\Omega_1} w_{it}\tilde{\tau}^*_{it}$

where Y_{it} is an outcome for unit i at time t, λ_i is a vector of unit-specific nuisance parameters, and δ is a vector of common covariates nuisance parameters.

The estimator in Callaway and Sant'Anna (2021) is defined below. Let $m^{group}_{g,t,\delta}(X)$ be population outcome regressions for the "not-yet-treated" group and define:

$$ATT^{group}_{dr}(g,t;\delta) = \mathbb{E}\left[\left(\frac{G_g}{\mathbb{E}[G_g]} - \frac{\frac{p_{g,t+\delta}(X)(1-D_{t+\delta})(1-G_g)}{1-p_{g,t+\delta}(X)}}{\mathbb{E}[\frac{p_{g,t+\delta}(X)(1-D_{t+\delta})(1-G_g)}{1-p_{g,t+\delta}(X)}]}\right)(Y_t - Y_{g-\delta-1} - m^{group}_{g,t,\delta}(X))\right]$$

where G_g is a binary variable for treatment in period g, $p_{g,t+\delta}$ is the probability of being treated in period $t+\delta$, $D_{t+\delta}$ is a binary variable for treatment status, and $Y_{i,t}$ is the observed outcome for unit i.

Finally, the estimator in Chaisemartin and D'Haultfoeuille (2020) is defined as:

$$\delta^S = \mathbb{E}\left[\frac{1}{N_S}\sum_{(i,g,t):t\geq 2, D_{g,t}\neq D_{g,t-1}} [Y_{i,g,t}(1) - Y_{i,g,t}(0)]\right]$$

where (i,g,t) is an observation, group, time triple, $D_{g,t}$ is the average treatment for a group time cell, $Y_{g,t}$ is average potential outcome, and N_S is derived using $N_{g,t}$, the number of observations in a cell.

Results using all three estimators are presented in the following section. All the estimators accommodate for binary treatment, which is a limitation on the results below. However, they all have different trade-offs. Chaisemartin and D'Haultfoeuille (2020) is the most general estimator and is used for continuous treatment results to supplement the set of binary treatment results. Borusyak et al. (2024) has the strongest assumptions on the control group, since it is used to create an imputation estimator. Callaway and Sant'Anna (2021) explicitly computes ATT by group and time, and handles covariates most rigorously, using a weighted group of controls.

4.3 Results

I use the sample of firms introduced in Section 3. The results in this section focus on a sample of firms that are all eventually treated: that is, the control group is the not-yet-treated group of firms. Figure 10 presents the benchmark result and Table 4 presents the analogous numerical estimates for the dynamic treatment effects. For the first 5 years after

loan receipt, the estimates and their confidence intervals grow larger over time, and indicate an increase in around 1 employee by the 5th period after loan receipt. The estimates for all three estimators are in the same magnitude.

Next, I consider how the estimates differ when varying the sample of firms by loan program. Figure 5 splits the sample by whether they participated in the 7(a) or 504 program. The estimates for firms in the 7(a) program are larger and have smaller standard errors. In contrast, the results for the 504 program indicate that the effects of the program are inconclusive in this setting. Table 6 presents the analogous estimates in tabular form.

Since the effects of the loan are positive for the 7(a) program, my next set of results specifically considers firms from this program. It is plausible that among this sample of firms, the effect of loan disbursement would vary along the dimension of firm characteristics, such as age, size, and loan amount. Figure 7 presents results split by firm age categories. The bins are firms of ages 1 to $\leq$ 4, 4 to $\leq$ 10, and 10 or more years respectively. The impact on employment is most positive for firms that are 4 years old or younger at the time of loan disbursement. Figure 8 presents results split by firm size categories. The bins are firms of size $\leq$ 5, 5 to 10, 10 to 25, and 25 or more employees respectively. The impact on employment is most positive for firms that have 5 employees or less. As firms become older and larger, the impact of the loan decreases.

Recent work by Haltiwanger et al. (2013) has shown that the relationship between firm size and employment growth is driven by firm age. To test if the results for firm size bins in this context is driven by firm age, I split firms into two bins by size (cutoff is 10 employees), and within those bins split them by age (cutoff is 5 years of age). Figure 9 presents results by both firm size and age bins. Within each row, the results of the figures are similar to one another, implying that firm age is not a strong determinant for the impact of loan receipt once firm size is already considered.

The benchmark results consider the loan as a binary treatment, which may plausibly be a weak assumption if the dispersion of loan amount as a fraction of net worth is high. While the estimators generally consider binary treatment, I address this concern by (i) splitting firm into bins by their loan to sales ratio, and (ii) using the continuous treatment estimator in Chaisemartin and D'Haultfoeuille (2020). Figure 10 presents results split by loan to sales categories. I measure the loan to sales ratio of firms in their year of treatment as the fraction of their gross approval amount to their reported sales in the event year. This measure is more informative than the loan amount by itself, as it takes into account a measure of the firm's expenses as well. The bins are split into quartiles of $\leq$ 0.14, 0.14 to 0.40, 0.40 to 1.43, and $\geq$ 1.43 respectively. Generally, the effect of loan receipt increases as the loan to sales ratio increases.

My second approach is to run the continuous treatment estimator in Chaisemartin and D'Haultfoeuille (2020), which presents an average cumulative effect per treatment unit of

1.71×10^{-6} (standard error of 7.24×10^{-7}) over 3.76 time periods of accumulated treatment effect. For a $400,000 loan, this estimate translates to a 0.684 increase in employment after almost 4 years, which is on the same magnitude as the benchmark estimates in Figure 10.

Finally, Figure 11 presents results for firms with loans of 3 years or less for the sample of firms in the 7(a) program. The estimates in years 4 and 5 are thus measuring the effect of the loan after it has already ended. Although the standard errors increase over time, the results indicate that the effect of the loan persists and continues to grow after it ends: in periods 4 and 5, the magnitude of the estimates continue to increase compared to previous periods. This result that will be directly referenced in the following section.

4.4 Identification

It is important to evaluate the robustness of the current empirical results to identification assumptions. The primary assumption is a conditional parallel trends assumption, in which absent loan receipt, the treated and not-yet-treated firms would have parallel employment trajectories, conditional on any controls. I make use of pre-trend tests for each of the estimators to prove that the assumptions behind a valid interpretation of estimates are satisfied. Table 17 presents a summary of assumptions that must be fulfilled by each of the estimators in order for the estimator results to be valid according to results provided in each of the papers, contained in the Appendix, Section A.2.

This assumption could fail if the timing of loan receipt and firm outcomes are endogenous. Although the exact timing of receipt is unknown to the firm, due to time spent on processing applications, the firms could plausibly expect an application being accepted after a few years, thus motivating firm-level decisions that impact firm outcomes even before the loan is received. Additionally, this assumption could fail if the timing of treatment itself is related to its growth trajectory, which would impact employment levels.

Figure 12 presents a test of the identification assumptions, in addition to the estimators' pre-trends tests that will be presented below. It takes the original sample of firms and adds (i) an event study period accounting for 1 year of anticipation effects, and (ii) baseline controls for sales and employment for the Callaway and Sant'Anna (2021) and Chaisemartin and D'Haultfoeuille (2020) estimators to control for firm sales and employment at first observation. In this way, the portion of the firm's growth trajectory related to baseline characteristics is accounted for. Although the pre-trends test does not pass for one of the estimators, in general the estimate magnitudes and confidence intervals are similar to the benchmark results in Figure 10.

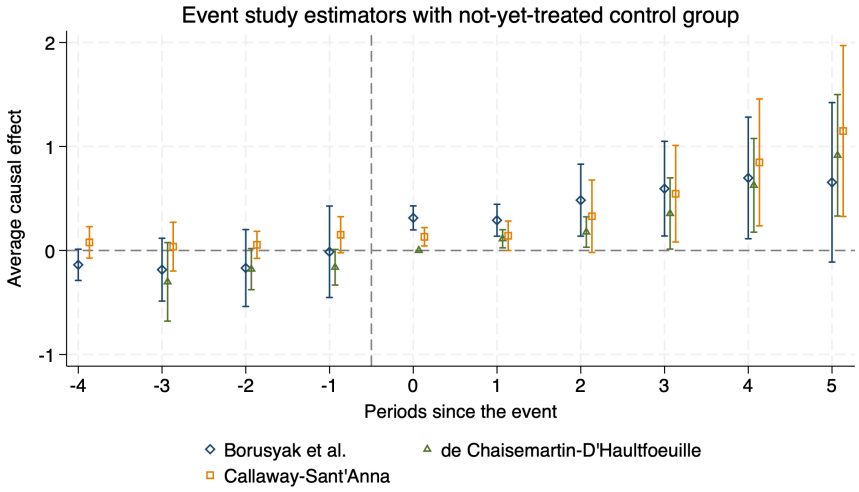
Tables 11, 12, and 13 present robustness results and pre-trends test results for the sample of firms that receive loans between 2011 and 2017. By construction, the not-yet-treated firms are used as the comparison group (rather than never-treated firms). Table 11 shows that the

aggregated treatment effect varies by cohort for the Callaway and Sant'Anna (2021) estimator, but the estimate is consistently positive. Moreover, the pre-trends test passes, with the null hypothesis that all pre-treatment coefficients equal zero not rejected. Table 12 presents pre-trend test results by horizon for the Borusyak et al. (2024) estimator. Whether or not the pre-trend test passes depends on the horizon: it passes for k=3 and k=4, but not for k=1 and k=2. Table 13 presents average treatment effect estimates and placebo estimator results for the Chaisemartin and D'Haultfoeuille (2020) estimator. All three placebo estimators are negative but statistically insignificant, and their joint test is not rejected. Therefore, the common trends assumption is not rejected by this test. In conclusion, the pre-trend tests show that Callaway and Sant'Anna (2021) and Chaisemartin and D'Haultfoeuille (2020) have parallel trends assumptions satisfied, while Borusyak et al. (2024) has its assumptions satisfied depending on the horizon used in the test.

4.5 Summary of Results

The results presented above measure the causal effect of loan disbursement on firm employment using heterogeneity-robust estimators. There is significant heterogeneity by program: the positive effects come from firms in the 7(a) program. The results using the sample of firms in the 7(a) program find that the average firm has approximately 0.5 to 0.8 more employees by 5 periods after loan receipt, which is approximately a 6.23% to 9.96% increase. Splitting the firms in the 7(a) program into bins by age and size, I find that it is the small firms driving the positive estimates. To address the binary nature of treatment, I also take sub-samples of firms by their loan-to-sales ratio, finding that firms with more borrowing relative to sales have larger treatment estimates. Finally, I focus specifically on firms with shorter loan lengths and find that the impact of the loan is persistent even after it ends.

Figure: Dynamic Treatment Effects on Employment



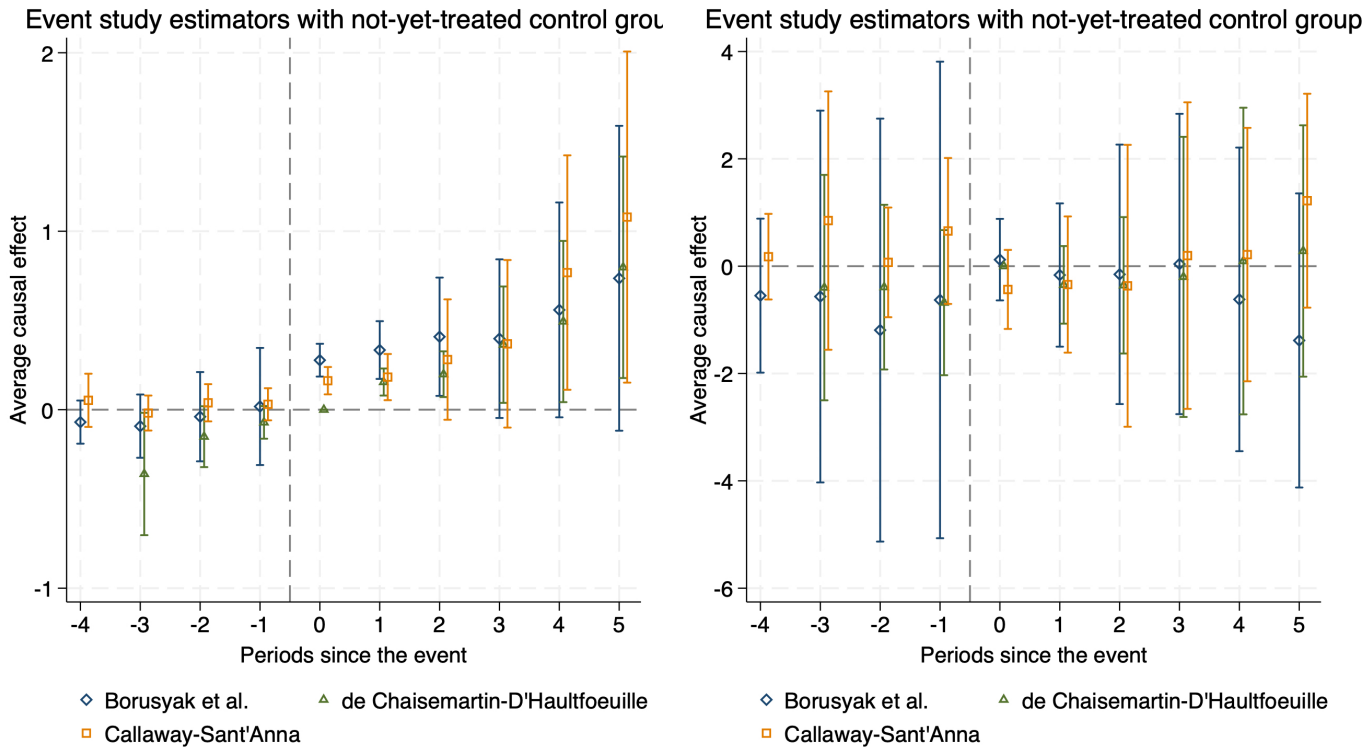
Note: This figure presents event study estimators using a group of treated firms (defined as receiving a loan in the years 2011-2018). The three estimators presented are the Borusyak et al. (2024) estimator, Chaisemartin and D'Haultfoeuille (2020) estimator, and Callaway and Sant'Anna (2021) estimator. Placebo coefficients are included for the estimators. The outcome is employment. The figure uses a sample of all firms.

Table 4: Dynamic Treatment Effect Estimates

	(1) CS	(2) B	(3) dCH
Period 1	0.120*	0.271*	0
	0.045	0.055	
Period 2	0.135	0.345*	0.113*
	0.072	0.084	0.045
Period 3	0.318	0.463*	0.150*
	0.18	0.18	0.072
Period 4	0.503*	0.577*	0.402*
	0.24	0.23	0.18
Period 5	0.777*	0.699*	0.634*
	0.31	0.30	0.23
Period 6	1.087*	0.644	0.983*
	0.42	0.39	0.30
Passes Pre-Trend Test?	✓	✓	✓

Note: This table presents dynamic treatment effect estimates for the three estimators, corresponding to Figure 10 part (a). * denotes statistical significance at the 5% level. Firms in the sample receive a loan in any year during the range 2011-2018, and the panel data is identified by firm-year.

Figure: Dynamic Treatment Effects By Program



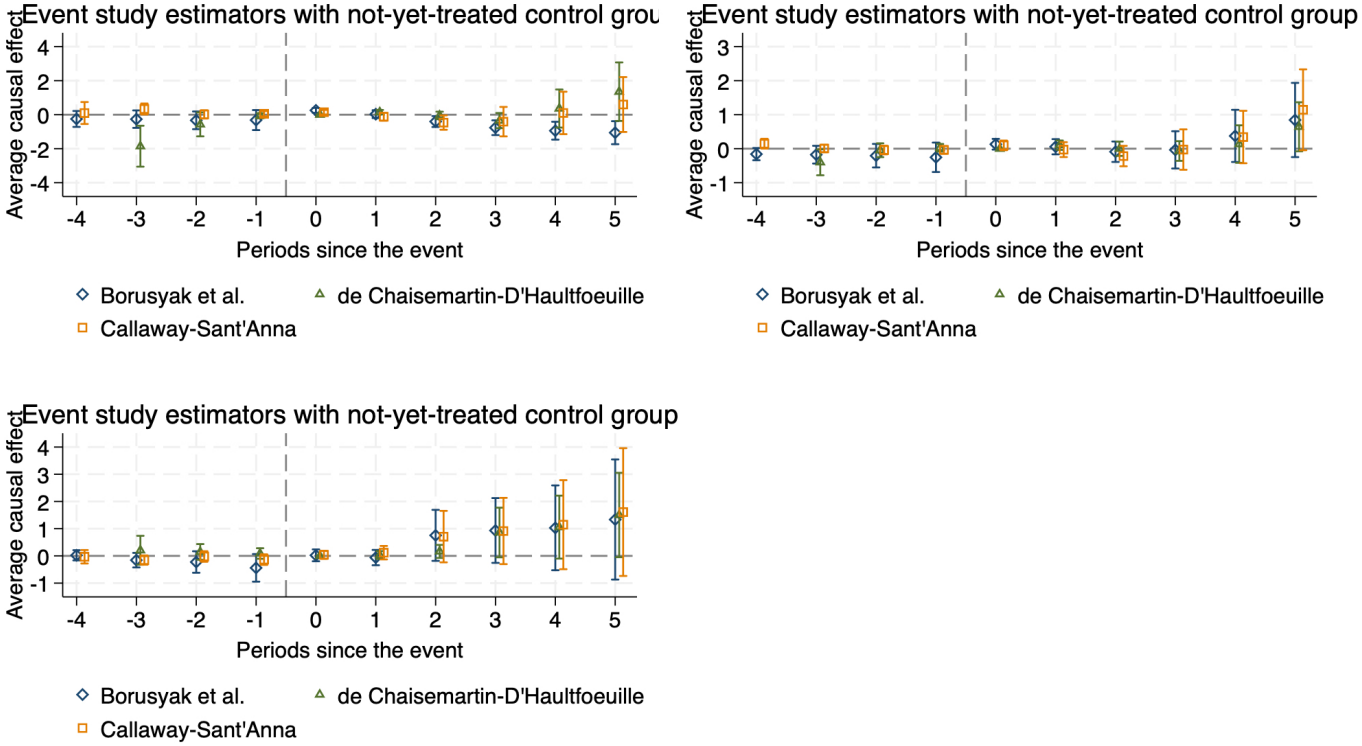
Note: This figure presents event study estimators for firms receiving a loan in any year during the range 2011-2018. The estimators presented are the Borusyak et al. (2024) estimator, Chaisemartin and D'Haultfoeuille (2020) estimator, and Callaway and Sant'Anna (2021) estimator. Placebo coefficients are included for the estimators. The outcome is employment. The left figure uses a sample of firms in the 7(a) loan program, and the right figure uses a sample of firms in the 504 loan program.

Table 6: Dynamic Treatment Effect Estimates

	7(a)			504		
	(1) CS	(2) B	(3) dCH	(4) CS	(5) B	(6) dCH
Period 1	0.163*	0.277*	0	-0.433	0.122	0
	0.039	0.047		0.38	0.39	
Period 2	0.183*	0.334*	0.155*	-0.342	-0.165	-0.349
	0.066	0.083	0.039	0.65	0.68	0.37
Period 3	0.281	0.409*	0.199*	-0.366	-0.152	-0.357
	0.17	0.17	0.065	1.34	1.23	0.65
Period 4	0.369	0.398	0.364*	0.197	0.0411	-0.200
	0.24	0.23	0.17	1.46	1.43	1.33
Period 5	0.768*	0.559	0.494*	0.216	-0.619	0.0955
	0.34	0.31	0.23	1.20	1.44	1.46
Period 6	1.079*	0.736	0.798*	1.220	-1.384	0.283
	0.47	0.44	0.32	1.02	1.40	1.19
Passes Pre-Trend Test?	✓	✓	✓	✓	✓	✓

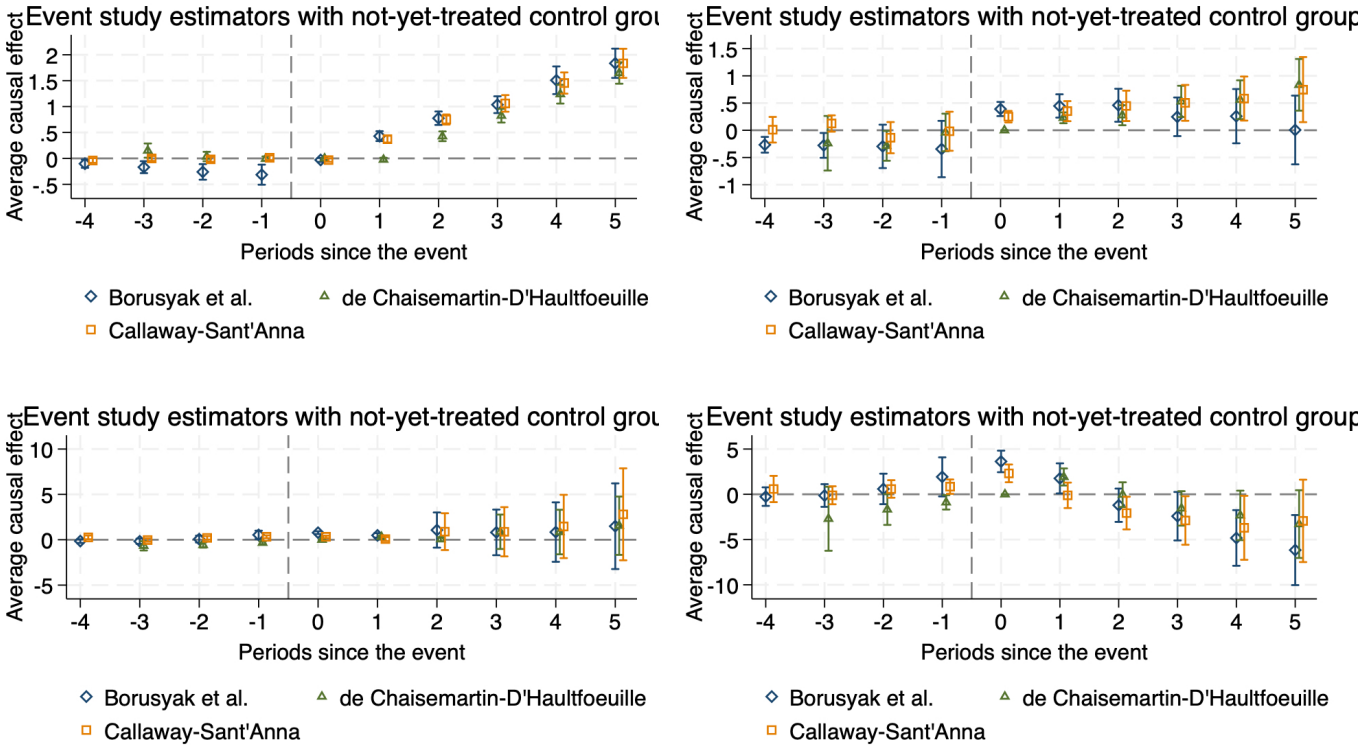
Note: This table presents dynamic treatment effect estimates for the three estimators, corresponding to Figure 5. * denotes statistical significance at the 5% level. Firms in the sample receive a loan in any year during the range 2011-2018, and the panel data is identified by firm-year.

Figure: Dynamic Treatment Effects on Employment, By Age



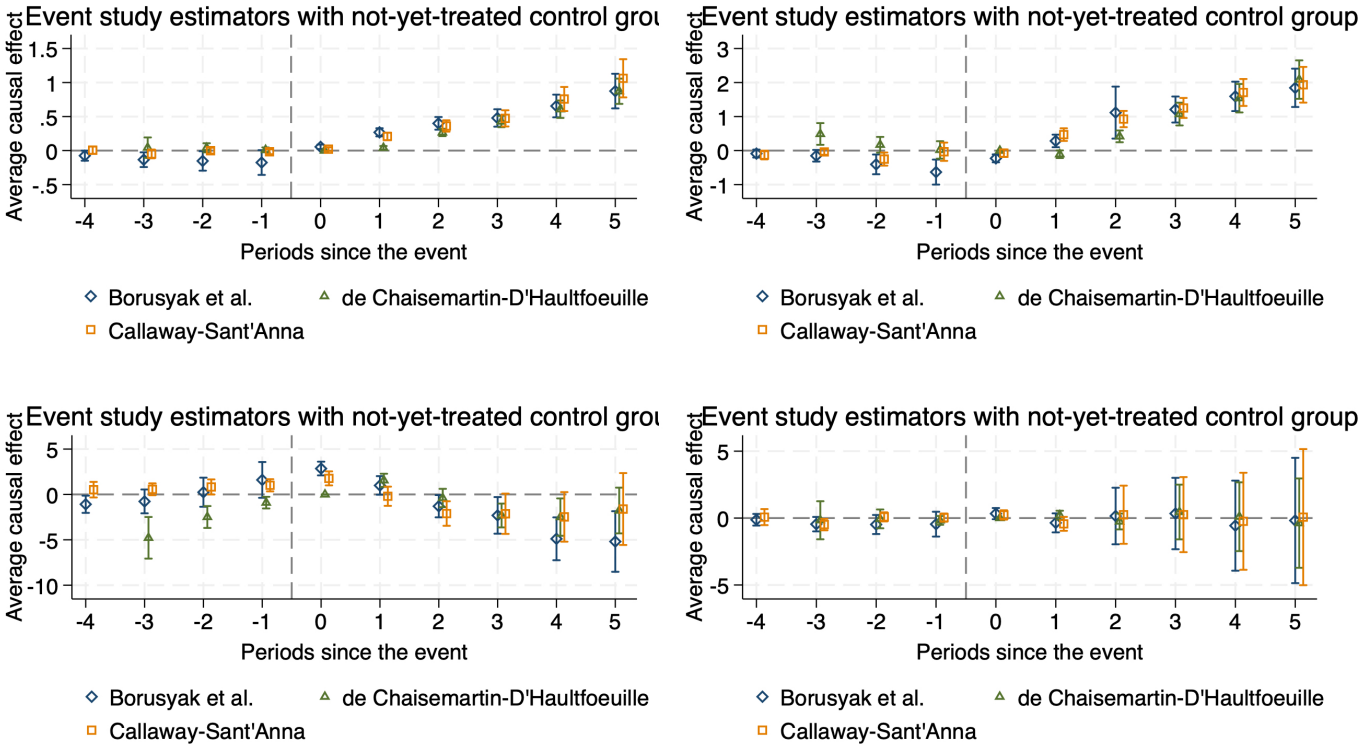
Note: This figure presents event study estimators for firms receiving a loan in any year during the range 2011-2018. The estimators presented are the Borusyak et al. (2024) estimator, Chaisemartin and D'Haultfoeuille (2020) estimator, and Callaway and Sant'Anna (2021) estimator. Placebo coefficients are included for the estimators. The outcome is employment and the sample is firms receiving a loan from the 7(a) program only.

Figure: Dynamic Treatment Effects on Employment, By Size



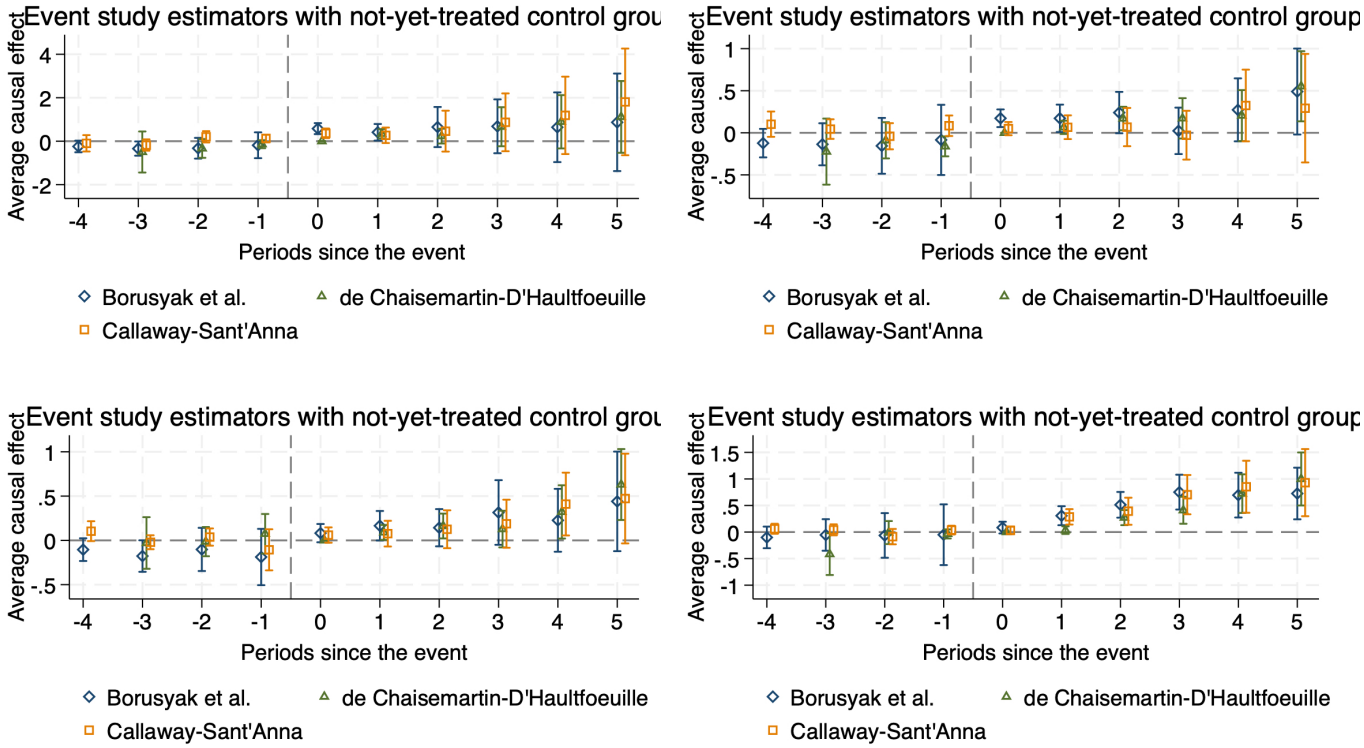
Note: This figure presents event study estimators for firms receiving a loan in any year during the range 2011-2018. The estimators presented are the Borusyak et al. (2024) estimator, Chaisemartin and D'Haultfoeuille (2020) estimator, and Callaway and Sant'Anna (2021) estimator. Placebo coefficients are included for the estimators. The outcome is employment and the sample is firms receiving a loan from the 7(a) program only.

Figure: Dynamic Treatment Effects on Employment, By Age and Size



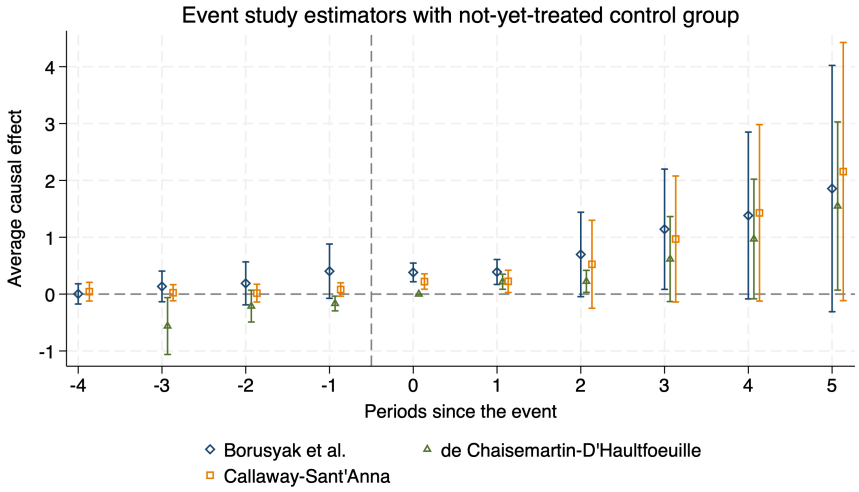
Note: This figure presents event study estimators for firms receiving a loan in any year during the range 2011-2018. The estimators presented are the Borusyak et al. (2024) estimator, Chaisemartin and D'Haultfoeuille (2020) estimator, and Callaway and Sant'Anna (2021) estimator. Placebo coefficients are included for the estimators. The outcome is employment and the sample is firms receiving a loan from the 7(a) program only.

Figure: Dynamic Treatment Effects on Employment, By Loan to Sales Ratio



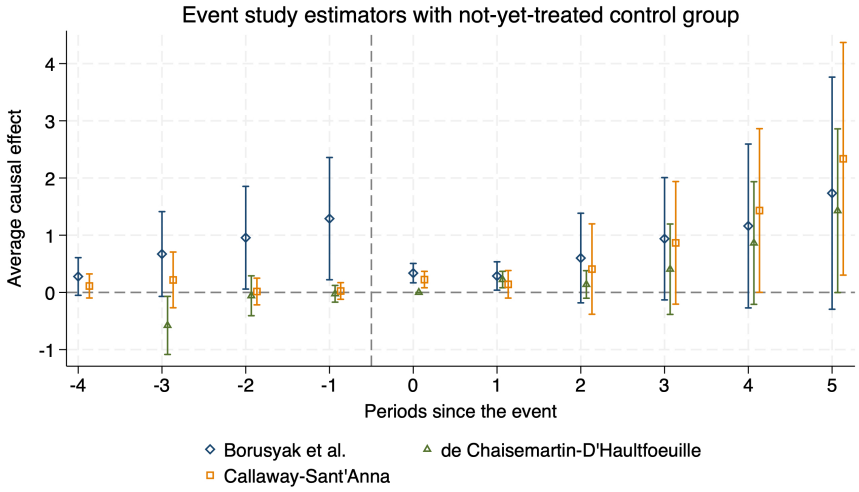
Note: This figure presents event study estimators for firms receiving a loan in any year during the range 2011-2018. The estimators presented are the Borusyak et al. (2024) estimator, Chaisemartin and D'Haultfoeuille (2020) estimator, and Callaway and Sant'Anna (2021) estimator. Placebo coefficients are included for the estimators. The outcome is employment and the sample is firms receiving a loan from the 7(a) program only.

Figure: Dynamic Treatment Effects on Employment



Note: This figure presents event study estimators using a group of treated firms (defined as receiving a loan in the years 2011-2018). The three estimators presented are the Borusyak et al. (2024) estimator, Chaisemartin and D'Haultfoeuille (2020) estimator, and Callaway and Sant'Anna (2021) estimator. Placebo coefficients are included for the estimators. The outcome is employment. The figure uses a sample of all firms with loan lengths of 3 years or less.

Figure: Testing Identification Assumptions on Benchmark Results



Note: This figure presents event study estimators using a group of treated firms (defined as receiving a loan in the years 2011-2018). The three estimators presented are the Borusyak et al. (2024) estimator, Chaisemartin and D'Haultfoeuille (2020) estimator, and Callaway and Sant'Anna (2021) estimator. Placebo coefficients are included for the estimators. The outcome is employment. The figure uses a sample of all firms.

Table 11: Callaway and Sant'Anna (2021) Aggregated Treatment Effect Estimates

	Partially aggregated							Single parameters
Simple weighted average								0.368*
								(0.13)
	g=2011	g=2012	g=2013	g=2014	g=2015	g=2016	g=2017	
Group-specific effects	0.365*	0.811	0.265	0.340*	0.0901	0.0450	0.0423	0.274*
	(0.18)	(0.49)	(0.20)	(0.16)	(0.13)	(0.13)	(0.096)	(0.099)
	t=2011	t=2012	t=2013	t=2014	t=2015	t=2016	t=2017	
Calendar-time effects	0.166	0.114	0.143	0.333	0.497*	0.467*	0.432*	0.307*
	(0.090)	(0.095)	(0.14)	(0.22)	(0.20)	(0.17)	(0.19)	(0.11)

Note: The table reports multiple aggregated treatment effect estimates under the unconditional parallel trends assumption as in Table 3 of Callaway and Sant'Anna (2021), using only not-yet-treated groups as the comparison. The pre-trend test under the null hypothesis that all pre-treatment coefficients are equal to 0 has a p-value of 0.1185, so it is not rejected at the 5% significance level. The standard errors are computed using asymptotic variance. * denotes significance at the 5% level.

Table 12: Borusyak et al. (2024) Pre-Trend Test Results

	k=1	k=2	k=3	k=4
Pre1	0.206	0.255	0.279	0.149
	(0.086)	(0.095)	(0.16)	(0.21)
Pre2		0.0569	0.0765	-0.0319
		(0.069)	(0.13)	(0.18)
Pre3			0.0215	-0.0643
			(0.11)	(0.15)
Pre4				-0.0910
				(0.070)

Note: This table presents OLS pre-trends test results as described in Test 1 of Borusyak et al. (2024), and the estimates are separate from the procedure measuring treatment effects according to the imputation estimator. For each column, the pre-trend is produced according to the horizon $k > 0$ that is specificed. The sample used for this test is not-yet-treated observations.

Table 13: Chaisemartin and D'Haultfoeuille (2020) Treatment Effects and Placebo Estimators

	Estimate	Standard error	Observations
$\hat{\beta}_{fe}$	0.25	–	–
DID_M	0.13	0.04	231073
DID_M^{pl}	-0.15	0.09	153779
$DID_M^{pl,2}$	-0.18	0.10	69465
$DID_M^{pl,3}$	-0.24	0.20	20255

Note: This table presents the DID_M and DID placebo estimators as defined in Chaisemartin and D'Haultfoeuille (2020), using only not-yet-treated groups as the comparison. The estimator DID_M^{pl} relies on a common trends assumption that can be tested by measuring if $\mathbb{E}[DID_M^{pl}] = 0$ holds. If the expectation is not 0, then "groups that switch treatment experience different trends before the switch than groups used to reconstruct their counterfactual trends when they switch."

5 Model

The previous section establishes a persistent and increasing effect on employment for up to 5 periods after loan disbursement. I now consider the SBA in a structural context as relaxing the collateral constraint. Below, I present a model of firm dynamics with financial frictions, and solve the model in steady-state equilibrium. Then, I run counterfactual exercises simulating the loan, and compute labor responses for treated firms, which are compared to the labor responses in empirical estimates to assess the validity of each counterfactual. I primarily test the loan as relaxing the collateral constraint (increasing credit access and implying financial constrainedness), and also test the loan as subsidizing capital rental payments.

5.1 Defining the Model

The model used is an extension of the Moll (2014) discrete time model with decreasing returns to scale (DRS) production. Under DRS production, every firm has a unique interior optimum for its factor demand choice given productivity level z. This aligns with the span of control argument in Lucas Jr. (1978) where each firm has fixed managerial talent.

There are two types of agents: entrepreneurs and workers. There is a unit measure of entrepreneurs and a unit measure of workers. To allow for entry and exit, entrepreneurs are subject to an exogenous death shock and are replaced by newborn entrepreneurs.

Workers are endowed with 1 efficiency unit of labor which they supply inelastically. They cannot save, so they are hand-to-mouth workers and can be omitted from the rest of the analysis.

Entrepreneurs have preferences:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_t \right] \tag{2}$$

Each entrepreneur owns a private firm which uses k units of capital and l units of labor to produce output according to:

$$y = f(z, k, l) = (zk)^{\alpha} l^{\gamma} \tag{3}$$

where α is the elasticity of output with respect to zk and γ is elasticity of output with respect to l, where $\alpha > 0$, $\gamma > 0$, and $\alpha + \gamma < 1$.

The logarithm of productivity z follows an AR(1) process:

$$\log(z_t) = \phi \log(z_{t-1}) + \varepsilon_t, \; \varepsilon_t \sim \mathcal{N}(0, \sigma^2) \tag{4}$$

Entrepreneurs hire workers in a competitive labor market at wage w_t and rent capital in a competitive capital rental market at rate $R_t = r_t + \delta$.

Entrepreneurs face collateral constraints:[6] [7]

$$k_t \leq \lambda a_t, \ \lambda \geq 1 \tag{5}$$

Define profit function:

$$\Pi(a_t, z_t) = \max_{k_t, l_t} \{f(z_t, k_t, l_t) - w l_t - (r + \delta) k_t\} \text{ s.t. } k_t \leq \lambda a_t \tag{6}$$

So their wealth evolves according to:

$$a_{t+1} = \Pi(a_t, z_t) + (1 + r_t) a_t - c_t$$

Entrepreneurs are indexed by their productivity z and wealth a, which are their state variables, with joint distribution $g(a, z)$, which is the state of the economy. The corresponding cumulative distribution function is $G(a, z)$. Newborn entrepreneurs draw their initial wealth levels from a uniform wealth distribution with a lower wealth cutoff of $\underline{a}$ and an upper wealth cutoff $\bar{a}$. They also draw their initial productivity level from the truncated stationary distribution of the AR(1) process in Equation 4.[8] [9]

The Bellman equation for the entrepreneur problem is:

$$V_t(a_t, z_t) = \max_{c_t} \log(c_t) + \beta \mathbb{E}_{z_{t+1}}[V_{t+1}(a_{t+1}, z_{t+1})] \tag{7}$$

$$\text{s.t. } c_t = \Pi(a_t, z_t) + (1 + r) a_t - a_{t+1} \tag{8}$$

A *competitive equilibrium* is defined as time paths for prices r_t, w_t and corresponding quantities, such that (i) entrepreneurs maximize utility subject to wealth law of motion taking as given equilibrium prices, and (ii) the capital and labor markets clear at each point in time:

[6] The microfoundation for a collateral constraint of this form is given in Kiyotaki (1998). The flow of funds constraint $c_t + x_t = y_t + \frac{b_{t+1}}{r_t} - b_t$ taken in conjunction with the debt repayment limit $b_{t+1} \leq \theta y_{t+1}$ results in a collateral constraint $x_t \leq \frac{\beta}{1 - \frac{\theta \alpha}{r_t}} a_t$ (given the definition $a_t = y_t - b_t$ and the result $c_t = (1 - \beta) a_t$, and where x_t is investment). In the case of default of the debtor-producer, the creditor can obtain a fraction θ of returns from production.

[7] The multiplicative collateral constraint form is standard in the recent literature. See Midrigan and Xu (2014) and Buera et al. (2011).

[8] Sub-section 5.3 presents details for the calibration of the initial conditions. $\underline{a}$ is chosen as the lowest level of the wealth grid, and $\bar{a}$ is defined as the percentile wealth of the upper wealth cutoff in the uniform distribution. $\bar{z}$ is defined as the cutoff for the truncated stationary distribution.

[9] Upon death, the firm's wealth is assumed to disintegrate so that the newborn firm receives only the asset level chosen from the uniform distribution with the calibrated upper cutoff. An alternative option is to take the firm's remaining capital as a lump sum rebate to workers.

$$\int k_t(a,z)dG_t(a,z) = \int a_t dG_t(a,z) = K_t$$

$$\int l_t(a,z)dG_t(a,z) = 1$$

A *steady-state competitive equilibrium* is a competitive equilibrium with $K_{t+1} = K_t$, $r_t = r$, and $w_t = w$.

5.2 Characterization of Equilibrium

Suppose that the economy is now in steady state. Entrepreneurs solve a static profit maximization problem period by period. They then split those profits between consumption and savings.

Solving for labor:

$$\frac{\partial \Pi}{\partial l} = (zk)^\alpha \gamma l^{\gamma-1} - w = 0$$

$$l^{\gamma-1} = \frac{w}{(zk)^\alpha \gamma} \Rightarrow \boxed{l^* = \left[\frac{w}{(zk)^\alpha \gamma}\right]^{\frac{1}{\gamma-1}} = \left[\frac{\gamma}{w} z^\alpha k^\alpha\right]^{\frac{1}{1-\gamma}}} \tag{9}$$

The optimal labor demand depends on the optimal capital demand, which varies based on whether or not the firm is constrained.

For an unconstrained firm, capital satisfies:

$$\frac{\partial \Pi}{\partial k} = \alpha z^\alpha k^{\alpha-1} l^\gamma - (r+\delta) = 0$$

$$\Rightarrow \alpha z^\alpha k^{\alpha-1} l^\gamma = r + \delta$$

Substituting the expression for labor from Equation 9 gives:

$$\alpha z^\alpha k^{\alpha-1} \left[\frac{\gamma}{w} z^\alpha k^\alpha\right]^{\frac{\gamma}{1-\gamma}} = r + \delta$$

$$\alpha z^{\frac{\alpha}{1-\gamma}} w^{\frac{\gamma}{\gamma-1}} \gamma^{\frac{\gamma}{1-\gamma}} k^{\frac{\alpha+\gamma-1}{1-\gamma}} = r + \delta$$

$$\boxed{k^u = \left[\left(\frac{\alpha}{r+\delta}\right)^{1-\gamma} \left(\frac{\gamma}{w}\right)^\gamma z^\alpha\right]^{\frac{1}{1-\alpha-\gamma}}} \tag{10}$$

If $k^u > \lambda a$, then $\boxed{k^c = \lambda a}$. The optimal capital choice is given by $k^* = \min(k^u, k^c)$.

Using the result from Equation 10, the optimal labor choice can then be written as:

$$l^* = \begin{cases} \left[\frac{\gamma}{w} z^\alpha (\lambda a)^\alpha\right]^{\frac{1}{1-\gamma}} & \text{if constrained} \\ \left[\left(\frac{\gamma}{w}\right)^{1-\alpha} \left(\frac{\alpha}{r+\delta}\right)^\alpha z^\alpha\right]^{\frac{1}{1-\alpha-\gamma}} & \text{if unconstrained} \end{cases}$$

Equation 7 presents the value problem for the entrepreneur. Recall that the consumption equation is:

$$c_t = \Pi(a_t, z_t) + (1+r)a_t - a_{t+1}$$

where

$$\Pi(a_t, z_t) = f(z, k^*, l^*) - wl^* - (r+\delta)k^*$$

$\Pi(a_t, z_t)$ can be further expressed in detail for the two possible solutions of capital demand. For the unconstrained solution, $\Pi(a_t, z_t)$ is:

$$\Pi(a_t, z_t) = (zk)^\alpha l^\gamma - wl - (r+\delta)k$$

$$= (z^\alpha)^{\frac{1-\gamma+\alpha\gamma}{1-\alpha-\gamma}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}} - w\left[\left(\frac{\gamma}{w}\right)^{1-\alpha} \left(\frac{\alpha}{r+\delta}\right)^\alpha z^\alpha\right]^{\frac{1}{1-\alpha-\gamma}}$$
$$-(r+\delta)\left[\left(\frac{\alpha}{r+\delta}\right)^{1-\gamma} \left(\frac{\gamma}{w}\right)^\gamma z^\alpha\right]^{\frac{1}{1-\alpha-\gamma}}$$

For the constrained solution, $\Pi(a_t, z_t)$ is:

$$\Pi(a_t, z_t) = z^{\frac{\alpha}{1-\gamma}} \lambda^{\frac{\alpha}{1-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} a^{\frac{\alpha}{1-\gamma}} - w\left[\frac{\gamma}{w} z^\alpha (\lambda a)^\alpha\right]^{\frac{1}{1-\gamma}} - (r+\delta)\lambda a$$

Consider the economy in steady-state equilibrium. For each productivity level z, there is a cutoff wealth level $\bar{a}_z$ below which all firms are constrained, and above which all firms are unconstrained. This productivity dependent threshold can be defined for each z:

$$\left[\left(\frac{\alpha}{r+\delta}\right)^{1-\gamma} \left(\frac{\gamma}{w}\right)^\gamma z^\alpha\right]^{\frac{1}{1-\alpha-\gamma}} = \lambda \bar{a}_z$$

So the market clearings can also be written:

$$\int_z \int_0^{\bar{a}_z} \left[\frac{\gamma}{w} z^\alpha (\lambda a)^\alpha\right]^{\frac{1}{1-\gamma}} d\psi(a) dg(z) + \int_z \int_{\bar{a}_z}^\infty \left[\left(\frac{\gamma}{w}\right)^{1-\alpha} \left(\frac{\alpha}{r+\delta}\right)^\alpha z^\alpha\right]^{\frac{1}{1-\alpha-\gamma}} d\psi(a) dg(z) = 1$$

$$\int_z \int_0^{\bar{a}_z} \lambda a d\psi(a) dg(z) + \int_z \int_{\bar{a}_z}^\infty \left[\left(\frac{\alpha}{r+\delta}\right)^{1-\gamma} \left(\frac{\gamma}{w}\right)^\gamma z^\alpha\right]^{\frac{1}{1-\alpha-\gamma}} d\psi(a) dg(z) = \int_z \int_a a d\psi(a) dg(z)$$

5.3 Calibration and Steady-State Equilibrium

There are 10 parameters in the model to calibrate: $[\alpha, \gamma, \beta, \delta, \phi, \sigma(z), \lambda, \chi, \bar{z}, \bar{a}]$. The first eight of these parameters are calibrated externally by drawing on values from the literature. The last two listed parameters are calibrated internally. Table 14 presents these parameters. The capital share (α), discount rate (β), and depreciation rate (δ) are taken from Moll (2014). The labor share (γ) is calibrated to 0.55, following estimations from the literature (see Atkeson and Kehoe (2005)) showing $\alpha + \gamma = 0.85$. The persistence (ϕ) and innovation (σ) components of the productivity process are calibrated using Karahan et al. (2024), where the parameters were internally calibrated within a group of 8 parameters by matching 23 moments that remained stable during the period 1979 to 2007.[10] The collateral constraint (λ) is calibrated using Beck et al. (2000). They measure the US external finance to GDP ratio to be 2.53, which implies a λ value of 4.15 in the model from Moll (2014) with constant returns to scale production. All firms in a given period are subject to an exogenous death shock (χ) at rate 5%.

Table 14: Calibration of Parameters

Equilibrium Parameters	Values	Source
Capital share (α)	0.3	Moll (2014)
Labor share (γ)	0.55	AK (2005)
Discount rate (β)	0.95	Moll (2014)
Depreciation rate (δ)	0.05	Moll (2014)
Productivity persistence (ϕ)	0.972	KPS (2024)
Productivity innovation ($\sigma(z)$)	0.032	KPS (2024)
Collateral Constraint (λ)	4.15	BDL (2000)
Death shock (χ)	0.05	BKS (2021)
Initial condition ($\bar{z}$)	0.91	Computed
Initial condition ($\bar{a}$)	0.44	Computed

Note: KPS (2024) stands for Karahan et al. (2024), BDL (2000) stands for Beck et al. (2000), AK (2005) stands for Atkeson and Kehoe (2005), and BKS (2021) stands for Buera et al. (2021). Initial conditions were computed using the method of simulated moments for the moments presented in Table 15.

To solve for the equilibrium, I search over candidate (r, w) price pairs. For each potential price pair, I calibrate the initial conditions. The initial conditions are calibrated to match realistic firm cohort life-cycle moments from the recent literature. In particular, I use firm size distribution statistics by age cohort from Cabral and Mata (2003) and growth rate

[10] Moments targeted in Karahan et al. (2024) are average startup size, conditional growth rates of three-year old firms by initial size, and exit rates of firms by age.

dispersion rates by age cohort from Decker et al. (2014).

Suppose that there are 2 parameters to calibrate: $\bar{z}$ and $\bar{a}$. $\bar{z}$ is the upper bound on a truncated stationary distribution of the AR(1) process in Equation 4. $\bar{a}$ is the percentile level on the wealth grid that serves as the upper cutoff on a uniform grid of initial wealth levels. $\underline{a}$ is 0.05 according to the defined wealth grid. Define $\theta = [\bar{z}, \bar{a}]$ as the vector of parameters.

I solve for θ using the method of simulated moments (MSM). Define $\hat{\theta}_{msm}$ as:

$$\hat{\theta}_{gmm} = \arg\min_{\theta \in \Theta} g(\theta)' W g(\theta) \tag{11}$$
$$g(\theta) = m^{data} - m^{sim}(\theta)$$

where W is the identity matrix, m^{data} is the vector of moments from the literature, and $m^{sim}(\theta)$ is the corresponding vector of moments from the simulated model.

Let $f \in \mathcal{F}_i$ denote each firm in some age cohort i. The size ratio between two cohorts of firms of ages i and j is defined as:

$$\text{Size Ratio} = \mathbb{E}[\log(l_{f,i})] - \mathbb{E}[\log(l_{f,j})]$$

The size dispersion of a firm cohort of age i is defined as:

$$\text{Size Dispersion} = \sqrt{\text{var}(\log(l_{f,i}))}$$

The growth rate of a firm cohort of age i, previously of age $i-1$, is defined as:

$$\text{Growth Rate} = \mathbb{E}\left[\left(\frac{l_{f,i}}{l_{f,i-1}}\right)\right]$$

Table 15 presents the moments used for calibrating 2 parameters: $\theta = [\bar{z}, \bar{a}]$. With an imposed lower bound of [0, 0.05] and upper bound of [1,1], the method of simulated moments (MSM) gives $\hat{\theta} = [0.9135, 0.4360]$, which results in simulated moments displayed in the table. All moments are computed using the definitions above.

The same draw of productivity shocks was used across all simulations.

Table 15: Selection of Moments

Moment	Literature Values	Simulated Moment	Source
Size Ratio (Ages 2-4 v. 10-19)	0.653	0.4907	Cabral and Mata (2003)
Size Ratio (Ages 5-9 v. 10-19)	0.282	0.1295	Cabral and Mata (2003)
Size Dispersion (Ages 2-4)	0.953	0.4555	Cabral and Mata (2003)
Size Dispersion (Ages 10-19)	1.092	0.3387	Cabral and Mata (2003)
Growth Rate (Ages 1-2)	1.118	1.2375	Decker et al. (2014)
Growth Rate (Ages 9-10)	1.018	1.0230	Decker et al. (2014)

Note: The parameters $\hat{\theta}$ were chosen through minimizing the expression in Equation 11.

Table 16: Prices

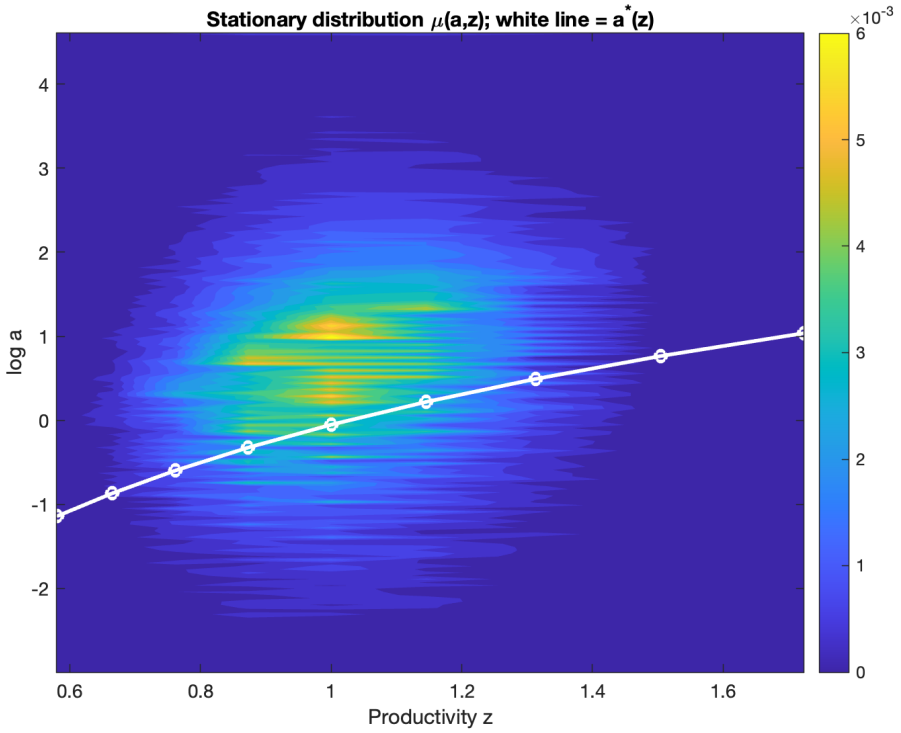
Equilibrium Parameters	Values
Wage (w)	0.76
Interest rate (r)	0.08

Note: Equilibrium prices were solved using the algorithm in Appendix A.3.

After calibrating the initial conditions, I solve for the numerical approximation of the joint distribution $g(a,z)$, which is used to calculate the excess labor and capital market clearings. I iterate over (r,w) pairs until both markets clear.

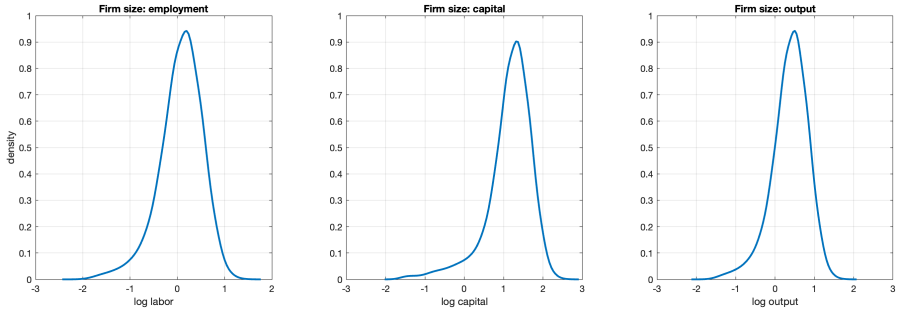
To complete the equilibrium, Table 16 presents the endogenous equilibrium prices w and r. In the steady-state equilibrium with parameters from Table 14, 26.77% of firms are credit constrained. A visualization of the state of the economy in steady-state is given by Figure 13, which also illustrates the threshold wealth value of constrainedness by productivity point. Figure 14 presents the steady-state distribution of firms by various outcome measures. Figure 15 presents the employment life-cycle profile of firms initiated according to the initial conditions.

Steady-State Joint State Distribution



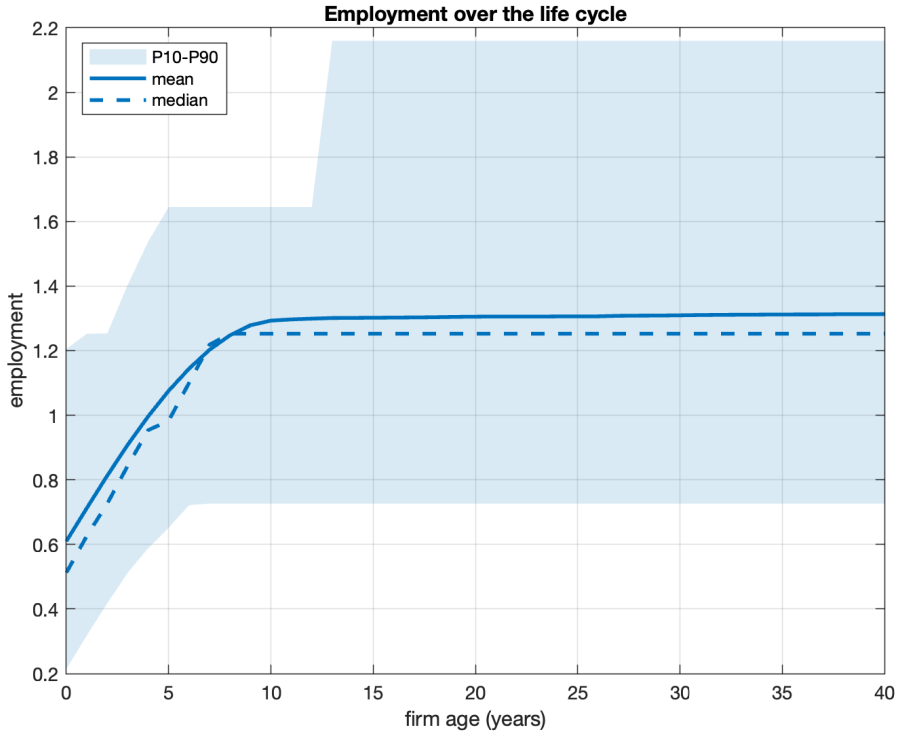
Note: This figure presents a visualization of $g(a,z)$, the joint distribution of the economy. It also includes a white line representing the threshold wealth value of constrainedness for each productivity point.

Steady-State Distribution by Firm Outcome



Note: This figure presents a visualization of firm density distributions by employment, capital, and output.

Steady-State Life-Cycle Diagram



Note: This figure presents a visualization of the life-cycle employment of a sample of firms simulated according to the initial conditions.

Firm simulations initiate firms according to AR(1) productivity shocks. The wealth grid is log-spaced with a minimum value of 0.05 and maximum value of 100: the minimum value ensures that firms with the lowest wealth levels are capital constrained. A large number of firms are first simulated with low initial a and z levels according to initial conditions. Productivity shocks for future periods are assigned using the Tauchen method,

and the wealth evolution is assigned according to the policy function a. Given the calibration, the value function problem is solved numerically to obtain the optimal policy function $a_{t+1}(a_t, z_t)$.

6 Using the Model to Interpret Effects of SBA Program

In this section, I assess the extent to which interpreting the SBA as expansion of access to credit is consistent with the empirical evidence from Section 3. The SBA is modeled as a counterfactual exercise where treated firms have a relaxed collateral constraint for the duration of the loan. After assessing the fit of the model to this interpretation of the SBA loan program, I then consider an alternative interpretation of the program as subsidizing the firm's rental interest payment.

The counterfactual exercise is implemented as follows. Newborn firms are drawn according to the initial conditions in steady state. After several periods, a group of firms are randomly selected to treatment with a loan. For each of these counterfactuals, I solve the new value function problem numerically, and use the new optimal policy function for the wealth evolution of treated firms. For non-treated firms, the wealth evolution is according to the original policy function.

The exercises are calculated modeling receipt of the loan as an MIT shock for the firm. For exercise 1, upon receipt of the loan, which is unexpected, firms then have the foresight that the collateral constraint λ_2 is relaxed for 1 period, marking the duration of the loan. Supposing that V^* is the value function in equilibrium, we solve for V^c as the value function for the counterfactual loan period:

$$V^c(a_t, z_t) = \max_{c_t} \log(c_t) + \beta \mathbb{E}_{z_{t+1}}[V^*(a_{t+1}, z_{t+1})] \tag{12}$$

The second counterfactual is implemented as a lump sum subsidy on the capital rental payment, specific to each firm depending on its capital level. The foresight of loan receipt is identical to the first exercise:

$$T(a_t, z_t) = (1-\tau)(r+\delta)k_t^*(a_t, z_t)$$

$$c_t = \underbrace{\Pi(a_t, z_t, T)}_{\pi(z_t, k_t^*, l_t^*) - wl^* - T} + (1+r_t)a_t - a_{t+1}$$

Average outcome measures (factor demands, output, wealth, etc.) are recorded for treated and control firms over time for up to 20 periods. The percentage difference in average labor demand for treated and control firms will then be compared to the benchmark event study estimates in Section 4. In particular, they are directly compared to the dynamic estimates in Figure 10.

The counterfactuals are done in partial equilibrium, such that prices remain the same as in steady-state as in Table 16. Since the SBA program represents a small fraction of total lending, implementing the counterfactuals is not expected to substantially impact general equilibrium prices. Hence, they are held fixed.

The target outcome computed in the counterfactual exercises is the *labor percentage increase*, which is defined as $\frac{\text{treated labor(t)-control labor(t)}}{\text{average firm size in period 0}} \times 100\%$. This outcome measure is analogous to $\frac{\text{coefficient estimate in period t}}{\text{average firm size in period 0}} \times 100\%$ from the data. Supposing that the counterfactual parameter is calibrated so that the magnitude of the labor percentage increase matches empirical estimates for one of the time periods, the trajectory of outcome measures across the model and data can be directly compared. As the trajectory of the treated firms' labor response is generated endogenously, comparing the shape of the response against empirical event study estimates serves as a validation test for each of the channels.

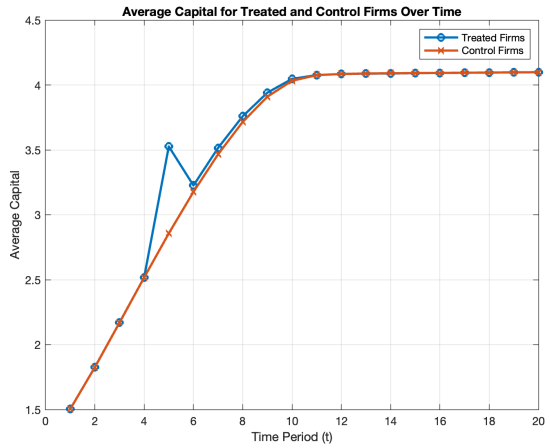
6.1 Loan As Relaxing Collateral Constraint

The results of the simulations are presented below considering the loan as a counterfactual that relaxes the collateral constraint.[11] Figures 16, 17, and 18 present results for Exercise 1, relaxing the collateral constraint on capital for treated firms. There is a noticeable increase in factor demand during the loan for treated firms, which returns to a similar level as control firms afterwards. The increase during the loan is driven directly by λ entering the factor demand expression for constrained firms, as shown in Equation 9. However, the increase during the loan and particularly afterwards is driven by a slight increase in wealth for treated firms, which remains elevated for several periods before returning to the same level as control firms. After T=10, the factor demands for both groups of firms are approximately equivalent due to most firms becoming unconstrained. The results match empirical estimates qualitatively: relaxing the collateral constraint results in an increased labor response compared to control firms, which is positive for several periods even after the loan ends.

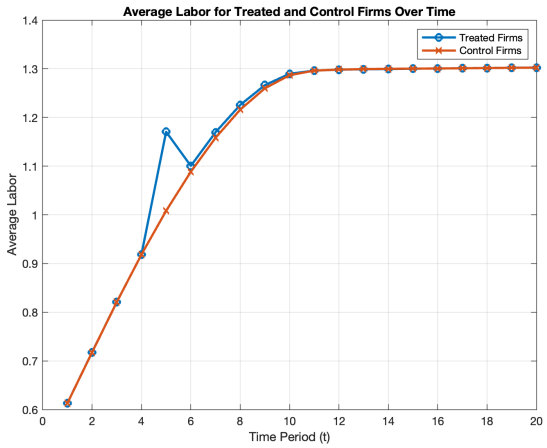
[11] Additional figures on factor demand dispersion and consumption are included in Appendix Section A.4.

Exercise #1: Average Factor Demands

(a) Average Capital



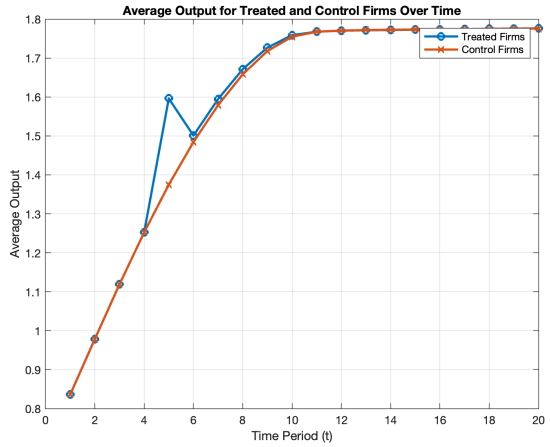
(b) Average Labor



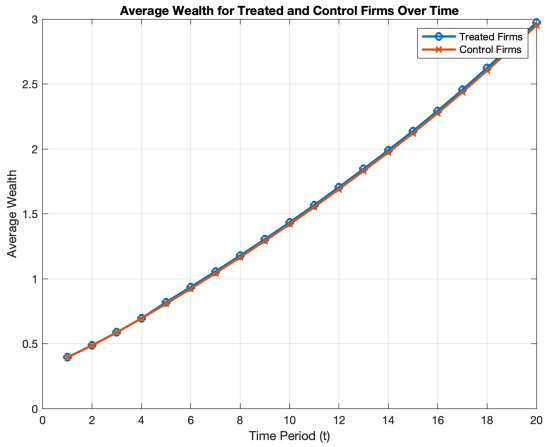
Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a $\lambda = 6$ for 1 period.

Exercise #1: Average Output and Wealth

(a) Average Output

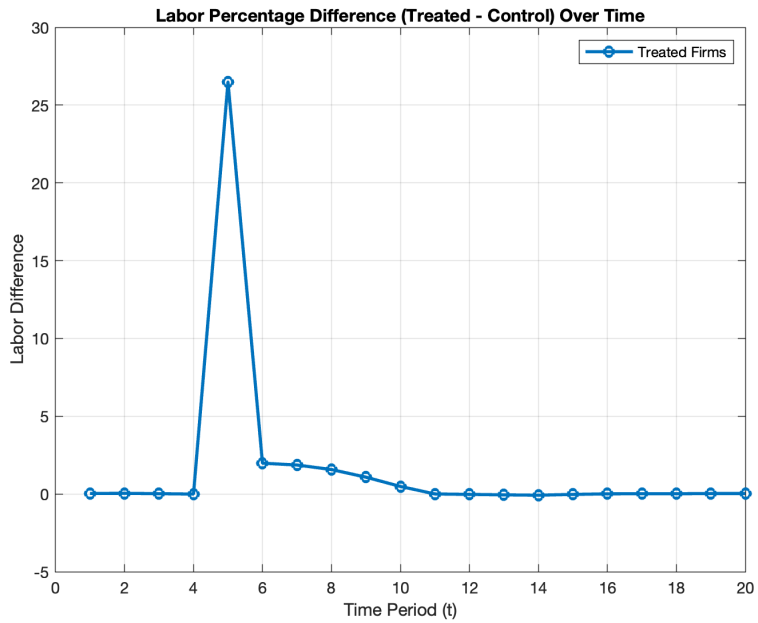


(b) Average Wealth



Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a $\lambda = 6$ for 1 period.

Exercise #1: Labor Percentage Difference



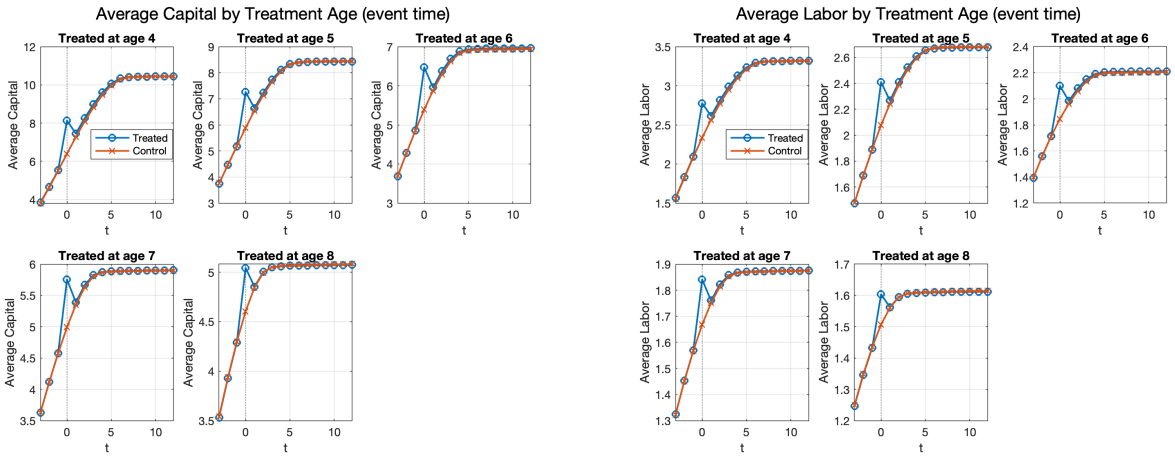
Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a $\lambda = 6$ for 1 period.

Next, I consider how the results depend on firm characteristics. Empirical results show that treatment responses to the loan vary based on firm characteristics such as size and age. In the context of the simulation, firm age is the cleanest dimension to use. Figures 19 and 20 present results following the same procedure as earlier results, but split into cohort of firms by age at treatment. Firms receiving the loan when younger have a larger treatment response, consistent with results in Section 4.

Exercise #1: Average Factor Demands, By Age

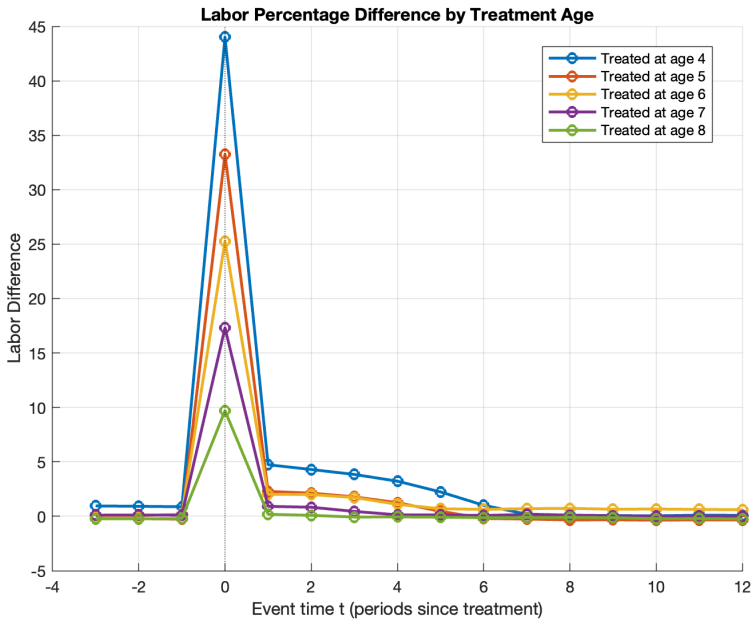
(a) Average Capital

(b) Average Labor



Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. The counterfactual is implemented for a randomly selected half of the firms which receive a $\lambda = 6$ for 1 period.

Exercise #1: Labor Percentage Difference, By Age



Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. The counterfactual is implemented for a randomly selected half of the firms which receive a $\lambda = 6$ for 1 period.

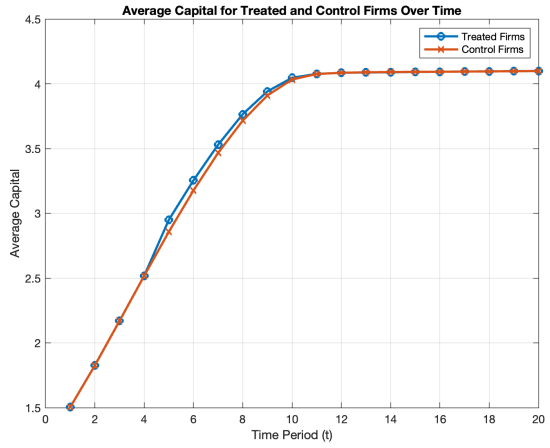
6.2 Loan As Subsidizing Capital Interest Payment

Next, I alternatively consider the loan as a counterfactual that subsidizes capital rental payments. Figures 21, 22, and 23 present results for Exercise 2, subsidizing the capital rental subsidy payment for treated firms. There is a slight increase in factor demands during the loan for treated firms, and the level remains elevated for several periods afterwards, gradually returning to the same level as control firms as both set of firms become unconstrained. The wealth trajectory for treated firms follows the same pattern.

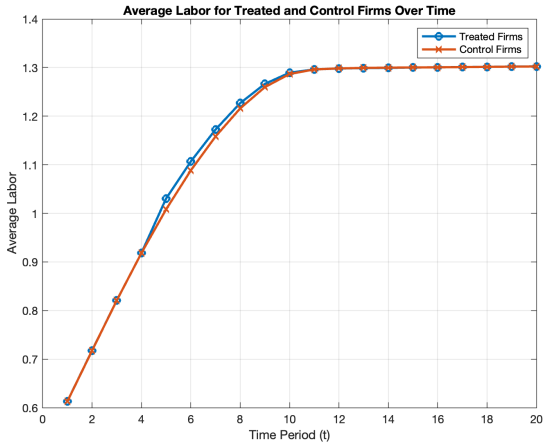
Figure 23 is a closer fit to empirical estimates than Figure 18. In Figure 18, the labor response during the loan spikes up, and then falls once the loan ends. While the labor response during the loan also spikes in Figure 23, the ensuing labor response gradually declines. This pattern is more aligned with empirical estimates from Figure 11 which shows that there is a persistent effect on employment after the loan terminates.

Exercise #2: Average Factor Demands

(a) Average Capital



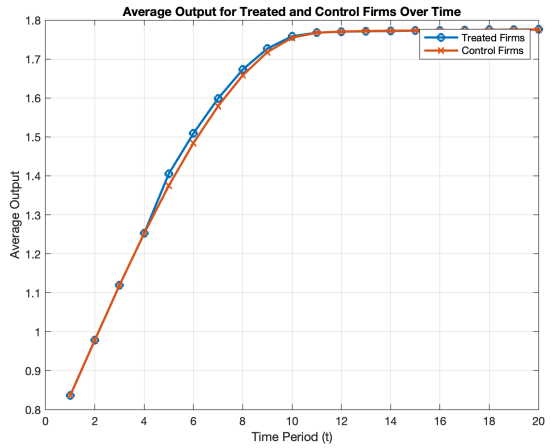
(b) Average Labor



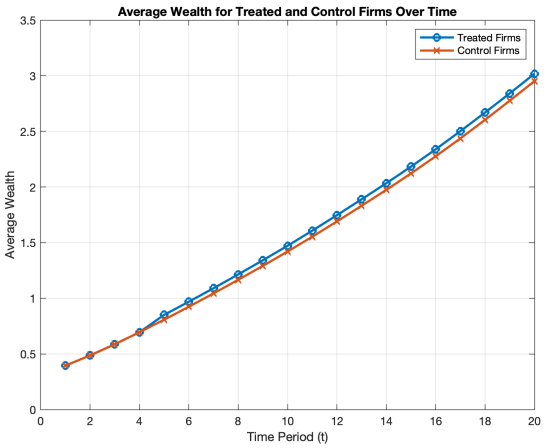
Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a 15% subsidy on capital rental payment for 1 period.

Exercise #2: Average Output and Wealth

(a) Average Output

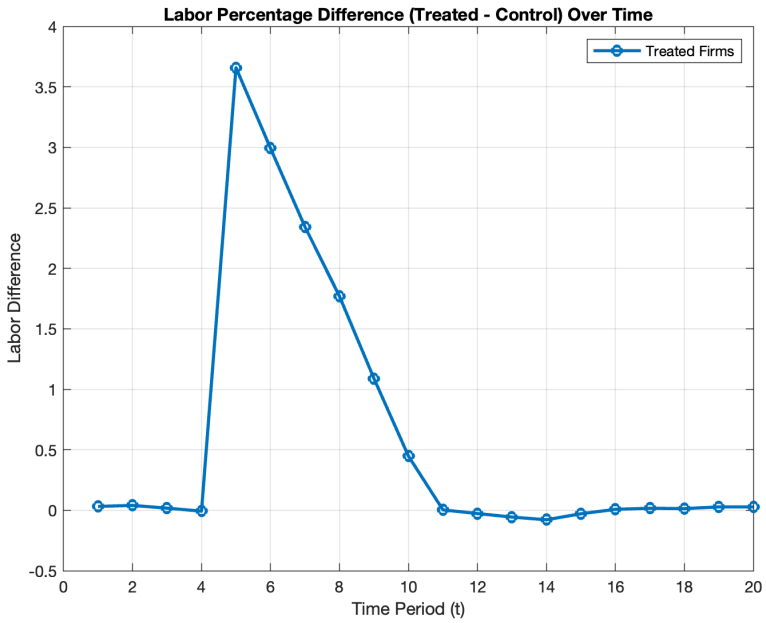


(b) Average Wealth



Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a 15% subsidy on capital rental payment for 1 period.

Exercise #2: Labor Percentage Difference



Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a 15% subsidy on capital rental payment for 1 period.

6.3 Analytical Results

Since the counterfactual is implemented for 1 period only and functions as a firm-level MIT shock, both treated and control firms have the same Euler Equation. To understand differences in factor demands after the counterfactual, I evaluate the difference in wealth holdings of treated and control firms after the loan under each counterfactual.

Define total wealth of the firm each period as $W(a_t, z_t; \lambda, \tau) = \Pi(a_t, z_t) + (1+r)a$, and define $a_{t+1} = \phi(W_t)$ as the function that maps the firm's total wealth to the amount that they save.

The object of interest is:

$$\phi(W_t(\theta_2)) - \phi(W_t(\theta_1))$$

where $\theta = \{\lambda, \tau\}$ is the parameter that is changed in the counterfactual.

This expression can be approximated using a first order Taylor approximation (this assumes differentiability of the optimal policy function):

$$\phi(W_t(\theta_2)) - \phi(W_t(\theta_1)) \approx \phi'(W_t(\theta_1)) \underbrace{(W_t(\theta_2) - W_t(\theta_1))}_{\text{computed below}}$$

The object $\phi'(W_t(\theta_1))$ is the derivative of the savings function at $W_t(\theta_1)$.

Exercise 1

- ***Case 1: Firm remains constrained***

$$W_t(\lambda_2) - W_t(\lambda_1) = \underbrace{z^{\frac{\alpha}{1-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} (\lambda_2^{\frac{\alpha}{1-\gamma}} - \lambda_1^{\frac{\alpha}{1-\gamma}})}_{C_1} a^{\frac{\alpha}{1-\gamma}} - \underbrace{(r+\delta)(\lambda_2 - \lambda_1)}_{C_2} a$$

$$\frac{\partial}{\partial a} = C_1 \frac{\alpha}{1-\gamma} a^{\frac{\alpha}{1-\gamma}-1} - C_2, \frac{\partial^2}{\partial a^2} = C_1 \frac{\alpha}{1-\gamma}\left(\frac{\alpha}{1-\gamma} - 1\right) a^{\frac{\alpha}{1-\gamma}-2} < 0$$

So the difference is concave, starting at 0 when the firm has no wealth. In this case, wl^* cancels out of the difference.

- ***Case 2: Firm transitions from constrained to unconstrained***

$$W_t(\lambda_2) - W_t(\lambda_1) = \underbrace{(f(z, k^*, l^*(k^*))}_{C_3} - f(z, \lambda_1 a, l^*(\lambda_1 a)))$$
$$- \underbrace{(r+\delta)k^* - wl^*}_{C_4} + (r+\delta)\lambda_1 a + wl^*(\lambda_1 a)$$

$$= (C_3 - C_4) - C_1 a^{\frac{\alpha}{1-\gamma}} + (r+\delta)\lambda_1 a + wl^*(\lambda_1 a)$$

$$\frac{\partial}{\partial a} = -C_1 \frac{\alpha}{1-\gamma} a^{\frac{\alpha}{1-\gamma}-1} + (r+\delta)\lambda_1 + wl_a(\lambda_1 a), \frac{\partial^2}{\partial a^2} = -C_1 \frac{\alpha}{1-\gamma}\left(\frac{\alpha}{1-\gamma} - 1\right) a^{\frac{\alpha}{1-\gamma}-2} > 0$$

Note that $\frac{\partial}{\partial a}\big|_{a=\bar{a}}$ can also be written as $\frac{\partial}{\partial a} = \lambda_1(-f_k + (r+\delta))$. From (i) diminishing marginal returns, and (ii) the firm being constrained ($k^* > \lambda a$), it follows that $f_k(\lambda a) > f_k(k^*) = r + \delta$ so $(r+\delta) - f_k < 0$ in this case, so the slope is negative. This region starts at $a = \frac{k^*}{\lambda_2}$ and ends at $a = \frac{k^*}{\lambda_1}$.

- ***Case 3: Firm remains unconstrained***

$$W_t(\lambda_2) - W_t(\lambda_1) = 0$$

This region starts at $a = \frac{k^*}{\lambda_1}$ and continues for the rest of wealth levels.

Exercise 2

- ***Case 1: Firm is constrained***

$$W_t(\tau_2) - W_t(\tau_1) = f(z, \lambda_1 a, l^*(\lambda_1 a)) - (1-\tau)(r+\delta)\lambda_1 a - f(z, \lambda_1 a, l^*(\lambda_1 a)) + (r+\delta)\lambda_1 a$$

$$= \tau(r+\delta)\lambda_1 a$$

The difference is linear in a, from $a = 0$ to $a = \frac{k^*}{\lambda_1}$.

- ***Case 2: Firm is unconstrained***

$$W_t(\tau_2) - W_t(\tau_1) = f(z, k^*, l^*(k^*)) - (1-\tau)(r+\delta)k^* - f(z, k^*, l^*(k^*)) + (r+\delta)k^*$$

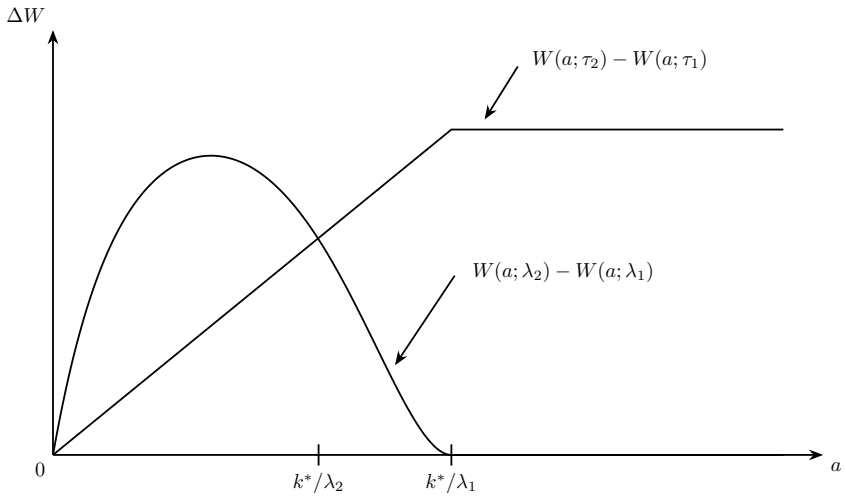
$$= \tau(r+\delta)k^*$$

For wealth levels $a = \frac{k^*}{\lambda_1}$ and higher, the difference is constant.

Comparison

Below is an illustrative diagram to show the differences in wealth for each counterfactual. The diagram is not drawn to scale. At lower wealth levels, treated firms in the first counterfactual have more total wealth than treated firms in the second counterfactual, compared to control firms at the given wealth level. However, at higher wealth levels, the opposite is true. In particular, after a firm becomes unconstrained, the collateral constraint parameter ceases to impact their total wealth levels, whereas they continue to receive a subsidy on rented capital, so their total wealth levels continue to be higher than if not treated.

ΔW Across Wealth Levels



Note: This figure, not drawn to scale, illustrates the difference between wealth levels of a treated and control firm for the two counterfactual exercises.

Additionally, exercise 1 has the effect that changing the counterfactual parameter directly leads to changes in the optimal factor demands.

- Case 1: The firm remains unconstrained both before and after the change. Their k^* and l^* do not change.
- Case 2: The firm is constrained under λ and unconstrained under λ_2. This means $\lambda a < k^* \leq \lambda_2 a$, so the firm increases their factor demands.
- Case 3: The firm remains constrained. Then the capital level increases from λa to $\lambda_2 a$.

7 Robustness of Results

In this section, I consider several alternative cases to evaluate the robustness of the results from the previous section.

Adjustment Costs

A natural question is to ask if the effect of loan receipt subsides directly after the loan is finished. Although the empirical estimates are only available for up to 5 years after loan receipt, estimating the results for a subsample of firms with shorter loan lengths suggests

that the employment effects of loan receipt persist even after the loan ends.[12] It would be thus reasonable to theoretically consider a case where the firm faces adjustment costs on labor.

$$\Pi(a_t, z_t, l_{t-1}) = \max_{k_t, l_t} \left\{ f(z_t, k_t, l_t) - w l_t - (r+\delta)k_t - \frac{\phi}{2}\frac{(l_t - l_{t-1})^2}{l_{t-1}} \right\} \text{ s.t. } k_t \leq \lambda a_t$$

$$V(a_t, z_t, l_{t-1}) = \max_{c_t} \log(c_t) + \beta \mathbb{E}_{z_{t+1}}[V(a_{t+1}, z_{t+1}, l_t)] \tag{13}$$

$$\text{s.t. } c_t = \Pi(a_t, z_t, l_{t-1}) + (1+r)a_t - a_{t+1} \tag{14}$$

$$\frac{\partial V(a_t, z_t, l_t)}{\partial l_t} = \frac{1}{c_t}\left(f_l - w - \phi\frac{l_t - l_{t-1}}{l_{t-1}} \right) + \beta \mathbb{E}\left[\frac{\partial V(a_{t+1}, z_{t+1}, l_t)}{\partial l_t} \right] = 0$$

$$\frac{\partial V(a_{t+1}, z_{t+1}, l_t)}{\partial l_t} = \frac{1}{c_{t+1}}\left(-\frac{\phi}{2} \right)\frac{l_t^2 - l_{t+1}^2}{l_t^2}$$

FOC for labor:

$$f_l = w + \phi\frac{l_t - l_{t-1}}{l_{t-1}} - \beta \mathbb{E}\left[\frac{c_t}{c_{t+1}}\left(-\frac{\phi}{2} \right)\frac{l_t^2 - l_{t+1}^2}{l_t^2} \right]$$

Under convex adjustment costs, partial adjustment has been proven by the prior literature (Hayashi (1982) for capital adjustment costs). When the firm exits steady-state and tries to expand its workforce, its optimal labor demand with adjustment costs will be below the frictionless optimal labor demand, and vice versa.

Although solving the model with adjustment costs is outside the scope of this paper, the question here is to what extent adding adjustment costs would change the results. Adding labor adjustment costs would dampen the current results for counterfactual exercise 1, but it remains that the mechanism behind the dampening is a friction, whereas the exercise 2 results come from an expansion in wealth.

Constant Returns to Scale

If the firm production function were constant returns to scale, a stronger version of the counterfactual results would hold. All firms would be constrained, and above the productivity threshold, operating firms would produce at their capital threshold. Therefore,

[12] As both my estimates and the existing estimates from the literature extend for a maximum window of 5 to 7 years after loan receipt, a promising direction for future research is to measure the effect of loan receipt for an extended window after the event.

λ limits factor demands for operating firms, and relaxing λ would increase their factor demands, without the buffer of firms already producing at their optimum.

For the second counterfactual, note that the optimal savings policy function is:

$$a_{t+1} = \beta[\lambda \max\{z_t \pi_t - r_t - \delta, 0\} + 1 + r_t]a_t$$

where wealth grows at a rate proportional to existing wealth. This rate is positive for all firms, and the rate increases for firms that receive a subsidy. After the subsidy concludes for treated firms, their wealth holdings continue to be higher than the wealth holdings of control firms, causing their labor demands to be higher as well. The effect can be expected to be more persistent than in the DRS case.

Borrowing Constraint Form

Suppose that the borrowing constraint were instead of the form $k \leq a + \bar{C}$, where $\bar{C}$ is a constant that may vary by firm. In this case, the analytical results carry through without change as long as $\bar{C}$ does not depend on wealth levels.

8 Conclusion

In this paper, I establish some new causal estimates of the effect of loan receipt on firm employment using heterogeneity-robust estimators and a staggered adoption event study design. I find a positive effect for up to 5 years after the loan, particularly for young and small firms in the 7(a) program. Next, I solve and calibrate a model of firm dynamics with financial frictions and interpret the effects of the program through the lens of the model. In particular, I examine the extent to which the loan could be modeled as relaxing a collateral constraint. An increased access to capital primarily acts through immediately raising firm factor demands, although this increase could be potentially muted with adjustment costs. Alternatively, a price of capital subsidy primarily impacts the firm through an increased accumulation in wealth over time. In the setting of the SBA and the model, I find that the persistence of the loan is captured more closely when it is interpreted as a reduction in capital. However, interpreting the loan as increased access to credit does match the empirics qualitatively and match treatment response facts based on firm age. Depending on the firm's initial wealth level, the loan impacts the firm differentially across wealth levels and across the two counterfactuals. The result is unlikely to be overturned by the extensions considered in the previous section, and is relevant to both the literature on firm dynamics with financial frictions and the literature on SBA loans.

There are several possible directions for future research. First of all, as new datasets become available, the results may be verified directly against firm data on loan applica-

tion acceptances and rejections. Second of all, while this paper is one of the first to test the collateral constraint channel of firm-level credit intervention, future research may test the robustness of the mechanisms more thoroughly to different model settings. Third, the counterfactuals done here can be used as a framework and applied to other empirical loan program settings to test the mechanism behind loan receipt.

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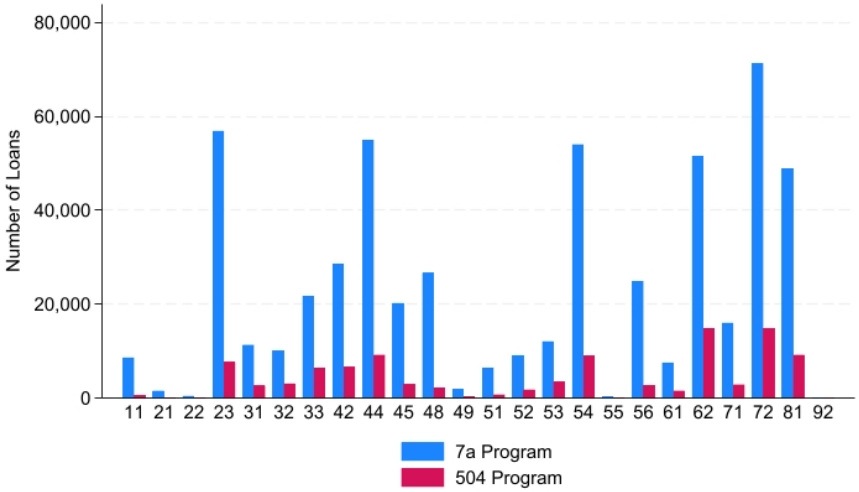
A Appendix

A.1 SBA Loan Data Cleaning

SBA loan data was accessed through the SBA Office of Capital Access website, and are accompanied by a data dictionary with descriptions for each variable.[13] The 7(a) and 504 loan data for FY2010-2019 were downloaded. For each of the two loan programs, the data were subject to punctuation cleaning of the `borrname` variable. Then, the data were merged annually with Dun and Bradstreet microdata to match firm loans to their corresponding outcome variables. Note that the Dun and Bradstreet data is establishment level, and loan data contains address information for each business, suggesting that it is also establishment level. We merge data based on establishment name and state, and choose the match based on the highest similarity score between street address variables. The four most commonly represented industries, ordered by North American Industry Classification System (NAICS) 2-digit sub-industries codes, are 23 (Construction), 54 (Professional, Scientific, and Technical Services), 72 (Accommodation and Food Services), and 44 (Retail Trade).

Figure 25 shows the distribution of loans from the two main programs by 2-digit NAICS code. 7(a) loans are more common than 504 loans, and across both loan programs, there is heterogeneity in number of loans by industry.

Figure: Total Loans by Industry



Note: This figure plots total number of 7a and 504 program loans by 2-digit NAICS codes from 2010 to 2019.

[13]The loan data were made available under the Freedom of Information Act (FOIA), which allows the public to access U.S. government documents.

A.2 Heterogeneity-Robust TWFE Estimators

Table 17: Summary of Estimators

Estimator	Testing Assumptions	Robustness
Chaisemartin and D'Haultfoeuille (2020)	The proposed ATE estimator δ^s is equal to DID_M under Assumptions 1, 2, 4, 5, 9-12. We use DID_M^{pl} to test the assumptions.	We can also use other placebo estimators such as $DID_M^{pl,2}$ and $DID_M^{pl,3}$.
Borusyak et al. (2024)	The imputation estimator τ_w needs Assumptions 1' and 2 to hold, which is tested with a robust OLS-based pre-trends test.	The OLS pre-trends test requires choosing the horizon k correctly to parametrize possible violations of the Assumptions.
Callaway and Sant'Anna (2021)	The conditional parallel trends assumption with never-treated or not-yet-treated group needs to be fulfilled.	Multiple ATT(g,t) estimates can be produced, such as by treatment year and by length of exposure. The conditional parallel trends assumption can be tested through conducting a joint hypothesis testing of pre-treatment estimates.

Note: This table presents descriptions of recent staggered adoption design estimators as they related to (1) testing the validity of assumptions, and (2) producing an array of estimates.

A.3 Equilibrium Price Calculations

The method to identify the equilibrium price pair (w, r) is through a 2D grid search. For each potential (w, r) pair, the discretized stationary joint distribution $g(a, z)$ is computed and the numerical approximation of labor and capital demand are computed. To ensure an invariant joint distribution, an exogenous death shock of 5% is imposed, and upon the death of a firm, a new firm is born according to the calibrated initial conditions.

Excess labor market and capital market demand are defined as:

$$L^{excess} \approx \sum_{i=1}^{N_a} \sum_{j=1}^{N_z} l(a_i, z_j)\hat{g}(a_i, z_j) - 1$$

$$K^{excess} \approx \sum_{i=1}^{N_a} \sum_{j=1}^{N_z} k(a_i, z_j)\hat{g}(a_i, z_j) - \sum_{i=1}^{N_a} \sum_{j=1}^{N_z} a_i\hat{g}(a_i, z_j)$$

using $\hat{g}(a, z)$ as the discretized probability distribution function approximating $g(a, z)$ for the N_a wealth grid points and N_z productivity grid points.

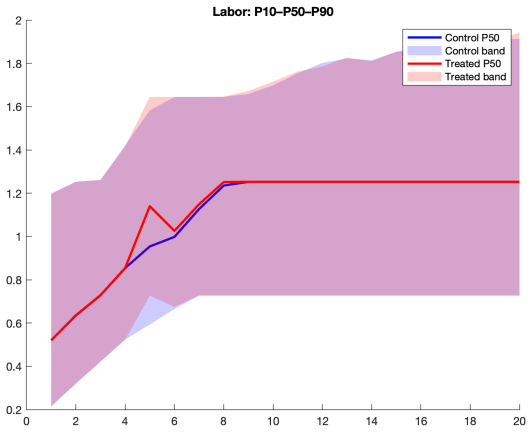
```
% define coarse grid of interest rates
% define grid of wages
% for each interest rate r
    % for each wage w, solve for excess labor demand
        % solve policy function given (r,w)
        % calibrate initial conditions for newborn firms given (r,w)
        % for iterations until convergence
            % simulate a large number of firms
            % compute a discretized g(a,z)
        % end
        % calculate excess labor market clearing
    % end
    % record labor market clearing wage
% end
% plot excess capital demand to identify region of crossing
% repeat procedure for a smaller grid of interest rates
```

From executing the grid search, the equilibrium price pair given the externally calibrated parameters in Table 14 is $(r, w) = (0,08, 0.76)$.

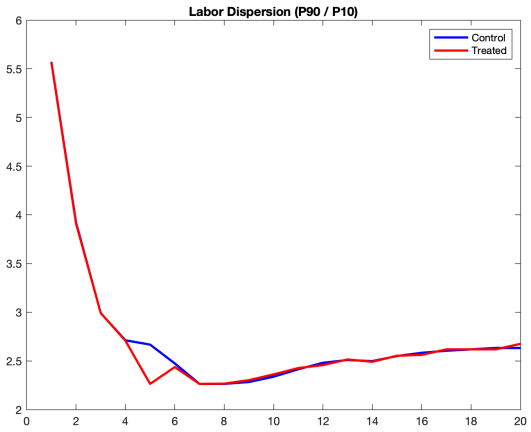
A.4 Counterfactual Exercise Additional Results

Exercise #1: Labor Demand Dispersion Measures

(a) Labor P10-P50-P90



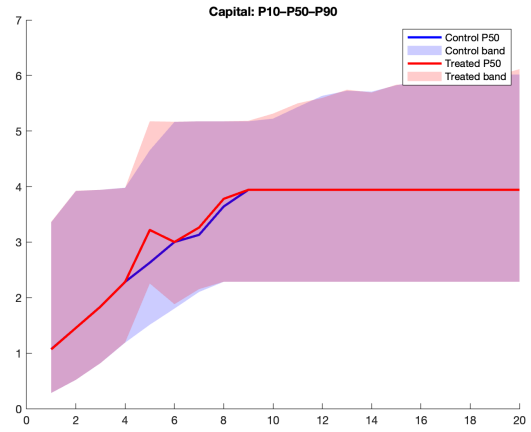
(b) Labor P90/P10



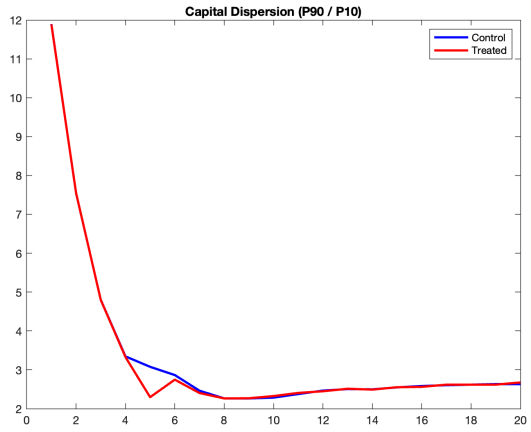
Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a $\lambda = 6$ for 1 period.

Exercise #1: Capital Demand Dispersion Measures

(a) Capital P10-P50-P90

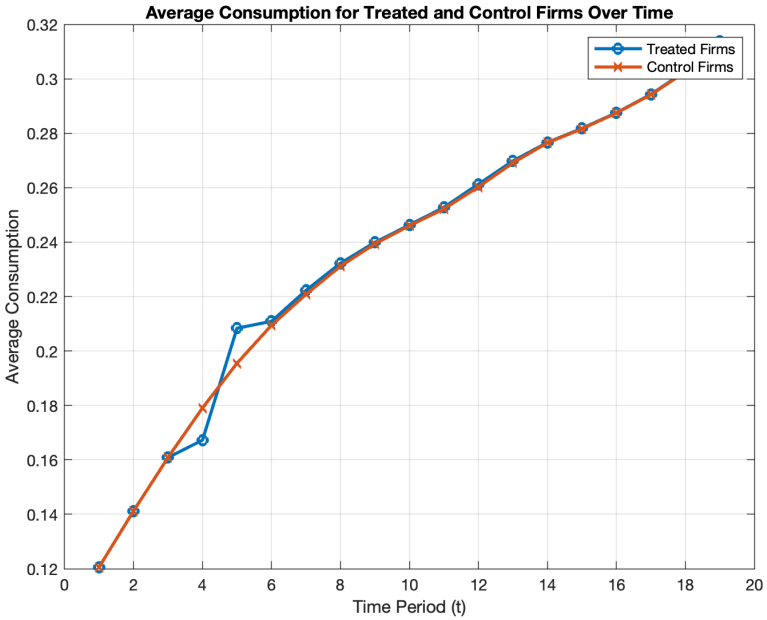


(b) Capital P90/P10



Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a $\lambda = 6$ for 1 period.

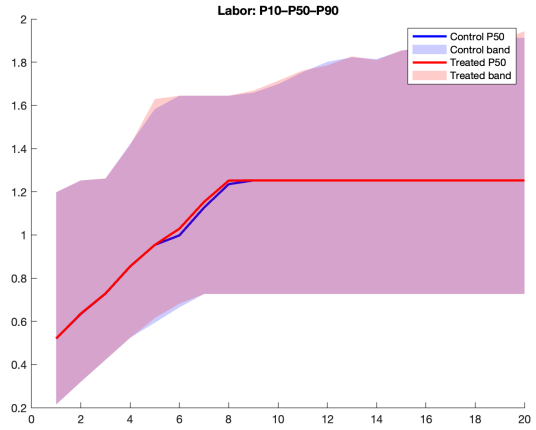
Exercise #1: Average Consumption



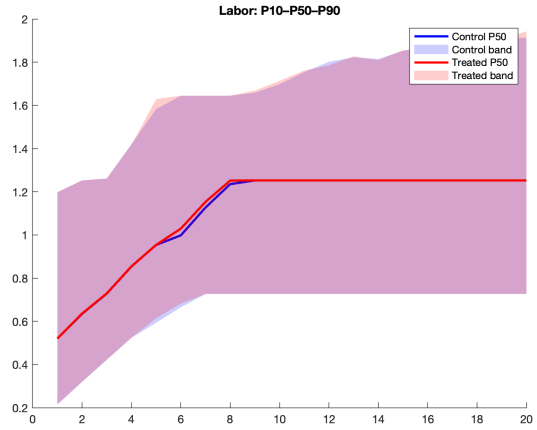
Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a $\lambda = 6$ for 1 period.

Exercise #2: Labor Demand Dispersion

(a) Labor P10-P50-P90



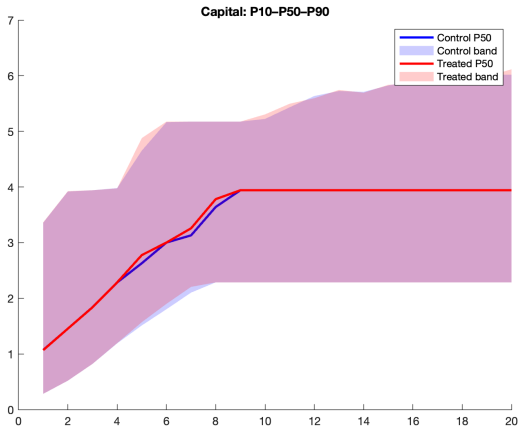
(b) Labor P90/P10



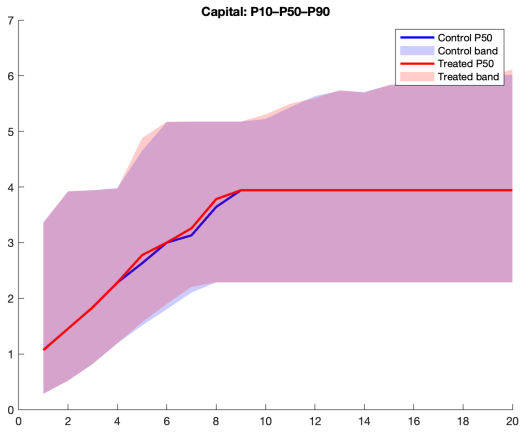
Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a 15% subsidy on capital rental payment for 1 period.

Exercise #2: Capital Demand Dispersion

(a) Capital P10-P50-P90

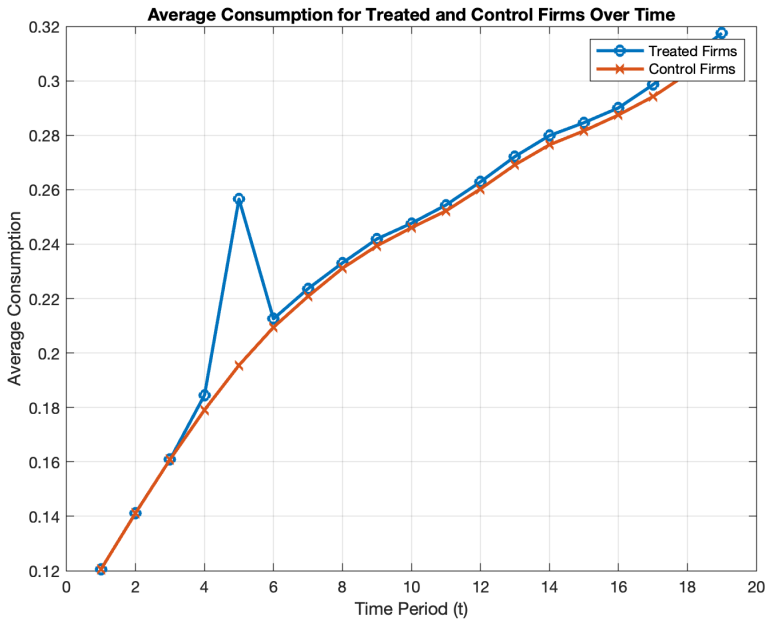


(b) Capital P90/P10



Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a 15% subsidy on capital rental payment for 1 period.

Exercise #2: Average Consumption



Note: This figure plots outcomes over T=20 for simulated firms. 20,000 firms were initiated according to initial conditions. At T=4, the counterfactual is implemented for a randomly selected half of the firms which receive a 15% subsidy on capital rental payment for 1 period.